SOME EXPERIMENTAL INVESTIGATIONS IN MATERIAL DAMPING

By

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INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
Lt R RAHMATHULLAH, Indian Navy

to the
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CERTIFICATE

This is to certify that the thesis entitled "Some Experimental Investigations in Material Damping" by Lt. R. Rahmathullah, I.N. is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

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NOMENCLATURE

b	Width of the specimen
C	Viscous damping coefficient
C_c	Critical damping coefficient
C_{eq}	Equivalent viscous damping coefficient
D_s	Energy dissipated by the system per cycle
D_v	Specific damping energy
d	Diameter of the holes
E	Modulus of elasticity of the material
f	Frequency at which the peak amplitude occurs
f_n	Fundamental frequency of vibration of the cantilever specimen
Δf	Difference of the two frequencies at which the amplitude is $\frac{1}{\sqrt{2}}$ times the peak amplitude
$\frac{\Delta f}{f}$	Half power band width
h	Height of the specimen
J, n	Damping constant and exponent of the material
J_1', J_2', n_1, n_2	Constants characterizing damping capacity of a material
K, K_u	Stiffnesses of the specimen with and without holes respectively
l_1	Distance of the centre of the strain gage from the free end of the specimen
l	Length of the specimen from the edge of the clamp to the free end

m	Mass of the system
P	Equivalent static load
V	Volume of the specimen
W_s	Maximum elastic energy stored by a specimen per cycle
W_v	Maximum elastic energy stored per unit volume of the material per cycle.
X_o	Amplitude of vibration
X_n, X_{n+1}	Two successive amplitudes of a free vibration record
X_{n+p}	Amplitude after p cycles from X_n
y	Distance of an element from the neutral axis
Δ	Tip deflection of the specimen with an end load P
δ	Logarithmic decrement
δ_u	Logarithmic decrement of the undisturbed (hole-free) specimen
ϵ	Harmonic strain amplitude
ϵ_m	Amplitude of measured strain
ϵ_x	Amplitude of strain in an element at a distance x from the free end
η_m	Material loss factor
η_s	Specimen loss factor
λ	Non dimensional hole size = d/b
σ	Harmonic stress amplitude

ψ	Specific damping capacity
ω	Frequency
ω_n	Natural frequency of the system
ζ	Damping ratio

SYNOPSIS

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The major objectives of this thesis are to develop a test-rig for measuring material damping and to use it to investigate the possibility of improvement of the damping capacity of structural members with introduced stress concentration and high damping inserts. Record of the free vibration of a cantilever specimen in vacuum has been used for evaluation of the material damping characteristics. The test-rig has been found to measure accurately the damping characteristics of even low damping materials like Aluminium. Damping constants of Aluminium, Cast Iron, Bakelite and Perspex have been evaluated.

Stress concentration in Cast Iron and Bakelite specimens has been introduced by drilling circular holes.

Improvement in the damping capacity of these specimens has been studied by using the test-rig. No significant improvement in the damping capacity has been observed in the case of the Cast Iron specimen. About 45% improvement in the damping capacity of the Bakelite specimens is obtained with an optimum size of the holes.

Inserts of Cast Iron, Bakelite and Perspex have been press-fitted into Aluminium strips to investigate the possibility of damping improvement by the use of high damping inserts. Cast Iron and Bakelite inserts have been found to give considerable improvement in the damping capacity of the Aluminium strip whereas there is no significant improvement with Perspex inserts. Change in the static rigidity of the specimen with holes and inserts has been found to be insignificant in all the cases.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Damping of a system is defined as the energy dissipation property of the system under cyclic loading. In structural members, the presence of adequate damping is often necessary to obtain suitable dynamic characteristics. Control of vibration and noise levels, specially at resonance, necessitates that the structural members be well damped. Most mechanical systems contain natural energy dissipation sinks at interfaces, joints, stress concentration areas and in the material itself. Artificial methods to increase the damping capacity of a system include the use of dash pots, interfacial slip damping, surface damping treatments etc. [1]*. There are situations, however, where the increment of the damping capacity cannot be achieved using the above artificial methods without hampering the functioning of the system. In such cases one has to depend on the inherent property of the material to dissipate the vibratory energy within itself. This energy is generally dissipated internally as heat [2]. Sometimes, however, a small part of the damping energy is absorbed

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*Numbers in box brackets designate References at the end of the text.

by internal structural changes that raises the internal energy level of the material.

There are certain materials which have a high damping capacity but low fatigue strength. Hence, a compromise has to be established between the two when using such materials for structures to be subjected to dynamic loading. Lazan [3] showed that the damping capacity of certain plastics is high enough to compensate for their low fatigue strength.

It has also been observed, from the properties of most of the available materials, that high damping capacity is possessed by those having poor strength and rigidity characteristics. Fortunately, the overall energy dissipation of a structure due to internal friction also depends on the design of the structure and its loading conditions. This is due to the dependence of the rate of energy dissipation on the magnitude of the dynamic stress at every point. This fact allows us to increase the damping capacity of a structure either by properly introducing high damping materials in small amounts or by suitably introducing stress concentration to change the overall stress distribution in the structure under the same loading condition. It should be noted that both these methods enhance the possibility of fatigue failure. Thus, these methods seem to be more suitable for structures designed on the basis of rigidity rather than strength considerations.

1.2 Short Review of Previous Work

Research on the damping properties of solid materials and their engineering significance started as early as 1784 when Coulomb conducted experiments and reported in his 'Memoir on Torsion' that the damping under torsional oscillations is not caused by air friction but by internal losses in the material. Till about 1950, many torsional vibration studies were undertaken on damping and 'internal friction' in metals and on the effect of such variables as stress amplitude, frequency, temperature, permanent strain, etc. and also to establish possible relationships between internal damping and other properties like fatigue strength.

Since 1950 damping and internal friction measurements on metals have become important as a tool for studying solid-state structures and micromechanisms. Recently the importance of damping as an engineering property, particularly in structures under dynamic loads, has motivated the scientific and engineering interest in the damping of polymers, elastomers, other non-metallic materials, and high-damping alloys.

In 1953, Lazan [4] showed that by introducing a proper stress distribution in a beam just by changing the shape of its cross section, the overall damping capacity of the beam can be increased. A considerable

amount of research has been conducted to evaluate the damping characteristics of members with brazed joints, notches and in Ball and Roller bearings [5].

Mallik and Ghosh [6, 7] have reported that the damping capacity of structural members can be improved by introduced stress concentration and by using high damping elastic inserts.

In reference [6], they considered the transverse vibrations of a cantilever strip with a series of holes drilled into it. They concluded that the method gives considerable increment in the damping capacity of the specimen only if the damping exponent (n) of the material is high. They also established the existence of an optimum diameter for the holes and the corresponding insignificant fall in the static rigidity.

In reference [7], the same strip was considered with a series of high damping elastic inserts in it. Two extreme cases were analysed, namely, welded inserts and press-fit inserts with zero interference. Theoretical analysis for different sizes and materials of inserts showed that

(i) both, welded and press-fit inserts considerably increase the damping capacity of structural members without any significant loss in static rigidity,

(ii) press-fit inserts perform better than welded inserts, and

(iii) there exists an optimum size of the annular inserts in case they are welded, but in case they are press-fitted, solid inserts give maximum improvement in the damping capacity.

With proper choices of insert materials, the increment in the damping has been found to be high enough so as to offset the effect of stress concentration resulting in higher dynamic load carrying capacity [8].

1.3 Objective and Scope of the Present Work

The objective of the present work can be enumerated under two headings:

(i) Development of a Test-Rig to measure accurately material damping which normally is of a very low order.

(ii) Experimental verification of the possible improvement of the damping capacity of structural members with introduced stress concentration and high damping inserts as analysed in references [6] and [7].

In the first phase of the work, a test-rig to measure the damping characteristics of a cantilever specimen has been developed. To minimize the effect of

air damping, arrangements have been made to vibrate the specimen in vacuum. An Aluminium specimen has been chosen for the pilot experiments to determine the capability of the test-rig. From the specimen damping characteristics, damping characteristics of four different materials, namely, Aluminium, Cast Iron, Bakelite,and Perspex, have been determined.

The next phase of the work consists of conducting experiments to evaluate the improvement of the damping capacity of specimens with introduced stress concentration and inserts, using the test-rig. Investigations on the improvement of damping capacity with introduced stress concentration have been carried out on specimens of two different materials, namely, Cast Iron and Bakelite. The improvement of the damping capacity has been expressed as a ratio of the logarithmic decrements of the specimens, with and without introduced stress concentration, measured at the same amplitude of the free end. The loss in the static rigidity of the specimen due to the holes has also been investigated.

Aluminium specimens, with inserts (both, solid and annular) of Cast Iron, Bakelite and Perspex, have been tested to investigate the improvement in the damping capacity and the change in the static rigidity. Here also, the ratio of the logarithmic decrements of

the specimens with and without inserts has been used to express the improvement in the damping capacity.

The limitations of the present work are as listed below.

(i) All experiments have been conducted covering low and intermediate ranges of strain.

(ii) Only the overall damping improvement is given and no attempt has been made to pin-point specific contributions from different mechanisms.

(iii) No attempt has been made to determine the change in fatigue strength when holes and inserts were used.

(iv) Equivalent static stress analysis has been used while evaluating the damping constants of the material from specimen characteristics.

CHAPTER 2

FUNDAMENTALS OF DAMPING

2.1 Different Indices for Damping Capacity

The nature of the sources of energy dissipation and associated mechanisms present in a system, are often different from one another. Hence the necessity arises to use different indices to specify the damping capacity of various systems [9]. The commonly used indices of damping and their inter-relationships are discussed below.

(i) Viscous damping coefficient: If the damping force present in a system is proportional to the velocity of oscillation, then the damping is said to be of viscous type and the proportionality constant is known as the viscous damping coefficient, commonly denoted by C.

(ii) Damping ratio: Often, the viscous damping coefficient is expressed as a non-dimensional quantity called damping ratio $\zeta = \frac{C}{C_c}$ where the critical damping coefficient $C_c = 2\, m\, \omega_n$ with m = mass and ω_n = natural frequency of the system. It should be noted that this definition is valid only for single degree of freedom systems. For multi-degree of freedom systems this concept can be used to express the modal damping when m will represent the modal mass and ω_n, the natural frequency of that particular mode.

(iii) Equivalent viscous damping coefficient: Even if the damping present in a system is not truly of viscous nature, it can be represented by an equivalent viscous damping coefficient for ease of subsequent mathematical analysis. The equivalence is obtained from the considera-tion of equal energy dissipation per cycle. The equivalent viscous damping coefficient, C_{eq}, is given by [10]

$$C_{eq} = \frac{D_s}{\pi \omega X_o^2} \qquad (2.1.1)$$

where D_s = energy dissipated by the system per cycle for harmonic oscillations with frequency ω and amplitude X_o.

(iv) Hysteretic damping coefficient: In many cases (particularly in cases of material damping) it is found that the energy dissipated per cycle of harmonic motion is independent of the frequency but proportional to the square of the amplitude. In such cases, one uses a hysteretic damping coefficient (h) and in the mathematical model, the viscous damping coefficient C is replaced by $\frac{h}{\omega}$ (for harmonic oscillations with frequency ω).

(v) Specific damping energy: Sometimes the damping capacity of a material is expressed by the energy dissi-pated by the material per unit volume per cycle of oscillation. This quantity is termed as the specific damping energy (D_v).

(vi) Specific damping capacity: This is a non-dimensional expression of the damping capacity of a material given by

$$\psi = \frac{D_V}{W_V} \qquad (2.1.2)$$

where W_V is the maximum elastic energy stored per unit volume of the material in that cycle.

(vii) Loss factor: The damping capacity of a specimen is also represented by a non-dimensional quantity called loss factor (η_s). If the energy dissipated by a specimen per cycle be D_s and the maximum elastic energy stored in that cycle be W_s then the loss factor is defined as

$$\eta_s = \frac{D_s}{2\pi W_s} \qquad (2.1.3)$$

Similarly the damping capacity of a material can also be defined in terms of the material loss factor (η_m) given by

$$\eta_m = \frac{D_V}{2\pi W_V} \qquad (2.1.4)$$

For linear visco-elastic materials, the loss factor defined by Eqn. (2.1.4) is also the ratio of the imaginary and the real parts of the appropriate elastic modulus [11].

(viii) Logarithmic decrement: This parameter defines the rate of decay of the free vibration of a

system and is given by

$$\delta = \ln \frac{X_n}{X_{n+1}} \tag{2.1.5}$$

where X_n and X_{n+1} are the two successive amplitudes of a free vibration record. For viscous damping, δ is independent of n and can also be expressed as [10]

$$\delta = \frac{1}{p} \ln \frac{X_n}{X_{n+p}} \tag{2.1.6}$$

where X_{n+p} is the amplitude after p cycles from X_n.

For free vibrations of a single degree of freedom system with small damping it can be shown that

$$\delta \simeq 2\pi\zeta \simeq \frac{D_s}{2 W_s} \simeq \pi \eta_s \tag{2.1.7}$$

(ix) Half power band width: Let for a single degree of freedom system, under harmonic excitation, f be the frequency at which the peak amplitude occurs and Δf be the difference of the two frequencies at which the amplitude is $\frac{1}{\sqrt{2}}$ times the peak amplitude. Then for small viscous damping, it can be shown that the quantity $\frac{\Delta f}{f}$, known as half power band width, is related to other expressions of damping as given below [10]

$$\frac{\Delta f}{f} \simeq 2\zeta \simeq \delta/\pi \tag{2.1.8}$$

2.2 Experimental Methods for the Measurement of Material Damping

The damping characteristics of a material can be determined both from free and forced vibration experiments. From the free vibration test, the decay rate of the amplitude is measured to yield an equivalent logarithmic decrement as defined by Eqns. (2.1.6) and (2.1.7). Material damping characteristics can thereafter be determined from this equivalent logarithmic decrement. This procedure has been used in the present work and will be elaborated in a subsequent chapter.

At steady state condition under forced vibration, the energy input per cycle must be dissipated by the specimen in that cycle. Thus, the energy dissipated by the specimen per cycle (D_s) can be directly determined by measuring the energy input per cycle to maintain steady state oscillations [12]. Another method of determining specimen damping characteristics from a forced vibration experiment is to measure the steady state amplitudes at various values of exciting frequencies. Then a graph of the amplitude versus frequency, known as the frequency-response curve, is plotted. Once this plot is obtained, one can use the half power band width index to obtain the damping capacity of the specimen [13]. For complicated structures, the half power band widths associated

with different modes can be conveniently obtained from a polar plot of amplitude and phase difference between the excitation and the response. This plot is known as the Harmonic Response Locus [14].

Once the specimen damping is obtained by one of these methods, the damping characteristics of the material can be determined subsequently.

CHAPTER 3

EXPERIMENTAL SET UP AND METHOD OF MEASUREMENT

3.1 Details of the Test-Rig and the Specimens

The experimental set up and the associated instrumentation are shown in Figs. 3.1 and 3.2. The set up uses a Perspex vacuum chamber fitted onto a vice housing made of mild steel. This housing was made rigid by embedding it in a concrete foundation. One end of the specimen was clamped in the vice. Care was taken to keep the clamping torque at a constant suitable value (50 ft. lbs) so as to ensure negligible slip damping at the vice. This will be discussed in detail in the next chapter. Care was also taken to keep the clamped length constant in all the cases. The vacuum chamber was evacuated by a mechanical vacuum pump and the vacuum created was measured by a mercury manometer. Free vibration of the specimen was initiated by means of an electro-magnet placed in the vacuum chamber, directly under the free end of the specimen. Ferro-magnetic strips were fixed onto the free ends of non-magnetic specimens. DC power to the electro-magnet was momentarily supplied and cut off to set the specimen in free vibration in its flexural mode. All connecting leads from the electro-magnet etc. were taken out through holes in the vacuum chamber. These holes, and all other

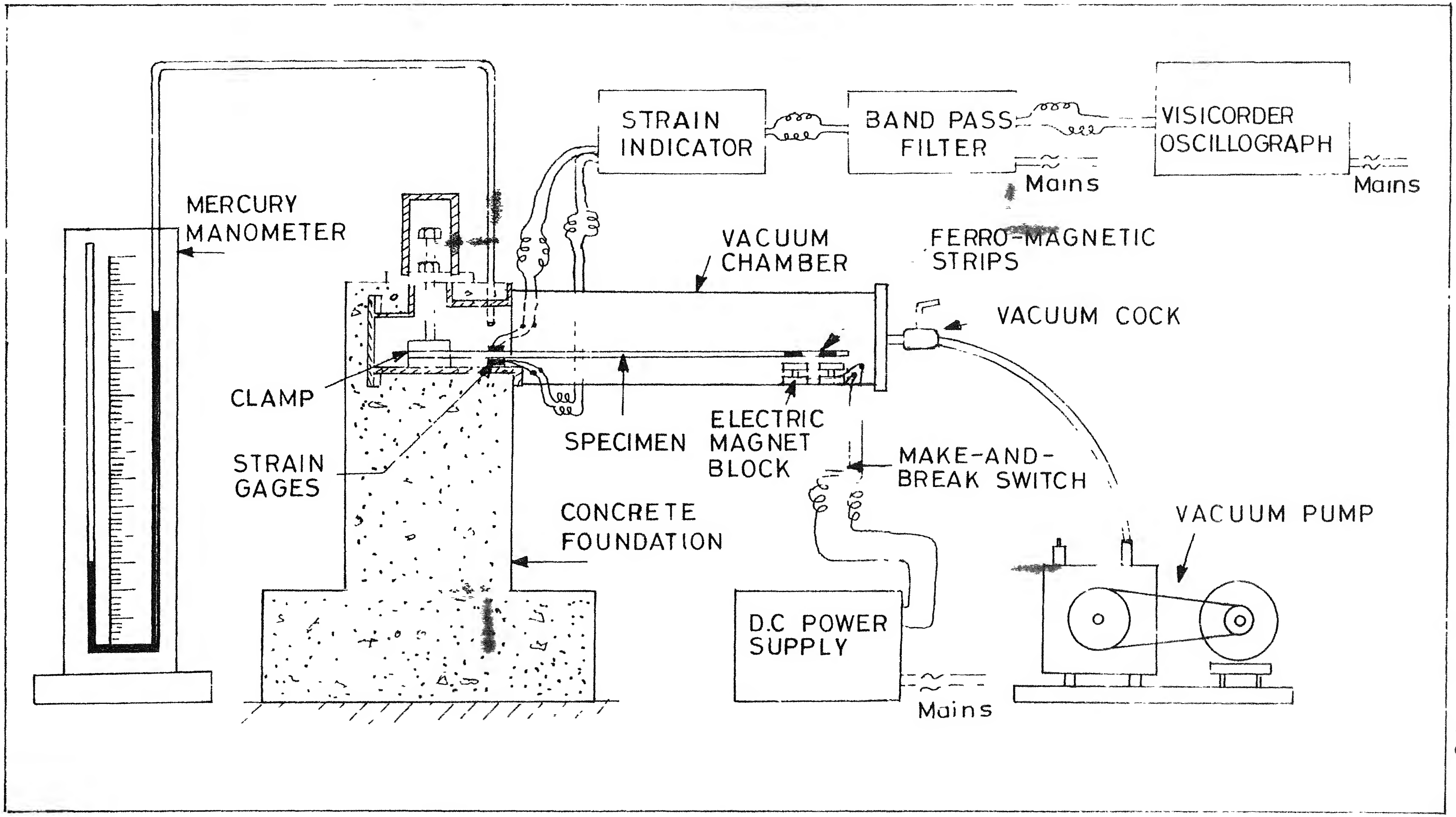


FIG. 3·1 EXPERIMENTAL SET-UP AND INSTRUMENTATION

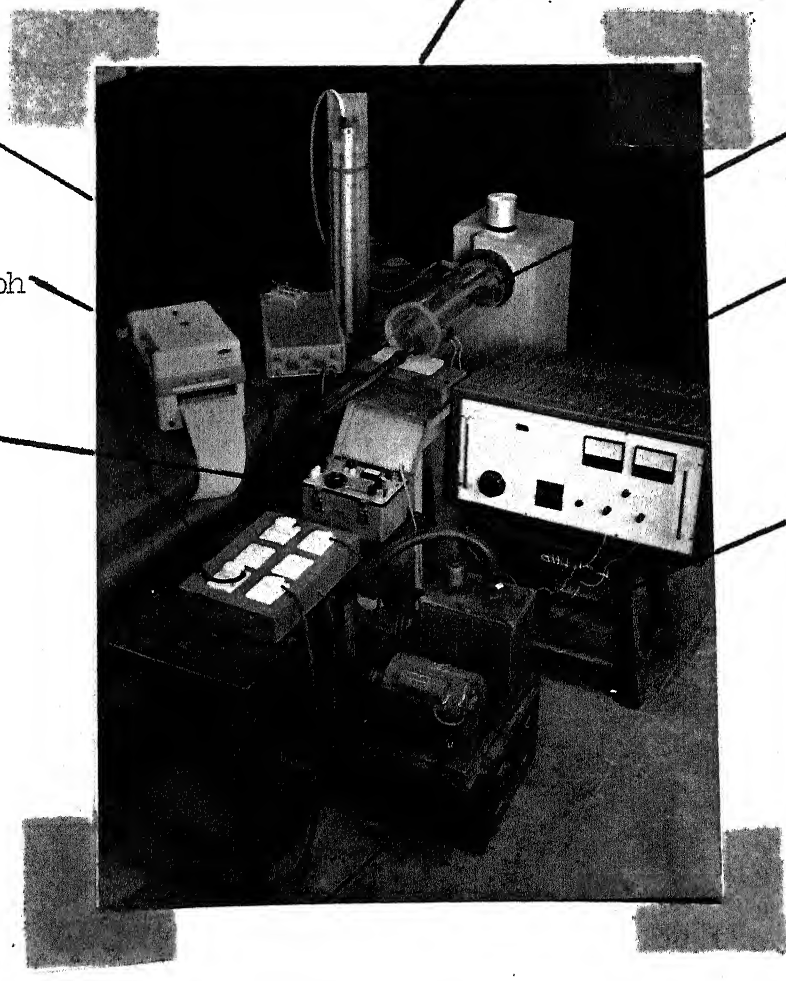


FIG. 3.2 EXPERIMENTAL SET UP

joints in the vacuum chamber were sealed by Apiezon vacuum grease.

Two strain gages were mounted on the top and bottom surfaces of all the specimens at a constant distance from the fixed end. These two gages automatically provided temperature compensation and also doubled the output signal. The gages were pasted according to the gage manufacturers' instructions. The details of the four basic specimens, used for measurement of the damping capacities of different materials, are listed in Table 3.1.

A study on the effect of stress concentration on the damping properties was conducted on Cast Iron and Bakelite specimens. Stress concentration in the specimens was introduced by drilling circular holes at a pitch of 1 in., starting at a distance of $\frac{1}{2}$ in. from the fixed end. Care was taken to ensure that the strain gages were pasted equidistant from the first two holes. This was done to avoid the effect of stress concentration on the gages and to ensure that the strain gages gave a measure of the tip deflection. Three hole sizes were tried in succession on both the specimens. The hole sizes used were 0.25 in., 0.4 in. and 0.5 in.

To study the effect of inserts on damping capacities of cantilever strips, Aluminium specimens with inserts of different materials were used. The

TABLE 3.1

DETAILS OF BASIC SPECIMENS

MATERIAL	E (lbf/Sq.in)	l (in)	b (in)	h (in)	l_1 (in)	f_n (Hz)
Aluminium	10.5×10^6	19.000	1.000	0.125	18.000	10.75
Cast Iron	15.0×10^6	13.780	1.000	0.072	12.780	8.25
Bakelite	4.0×10^6	17.625	1.000	0.125	16.625	6.75
Perspex	0.45×10^6	16.000	1.000	0.250	15.000	10.00

where E = modulus of elasticity of the material,

l = length of the specimen from the edge of the clamp to the free end,

b = width of the specimen,

h = height of the specimen,

l_1 = distance of the centre of the strain gage from the free end of the specimen,

and f_n = fundamental frequency of vibration of the cantilever specimen.

overall dimensions of these specimens were identical to those of the Aluminium specimen mentioned in Table 3.1. First, holes of 1/4 in. diameter were drilled on the centre line, along the length of the specimen. The centre distance between two successive holes was 1 in., the first hole being at a distance of 1/2 in. from the fixed end. Care was taken to mount the strain gages equidistant from the first two holes. Cast Iron inserts, of thickness equal to the specimen thickness, were pressed into the holes of one specimen, Bakelite inserts into another and Perspex inserts into the third. After the experiments and computations to evaluate the damping properties of these specimens were completed, concentric holes of 1/8 in. diameter were drilled in all the inserts thus making them annular. Figure 3.3 shows a sketch of an Aluminium specimen with annular inserts. These specimens were tested again for their damping characteristics.

In all, experiments and computations were conducted on a total number of 16 specimens, the results of which will be discussed in a subsequent chapter. Figure 3.4 shows a photograph of a few of these specimens.

3.2 Instrumentation and Method of Measurement

Figure 3.1 also shows the instrumentation used in the experiment. The leads from the two SR-4 strain gages were connected in a half bridge circuit incorporated

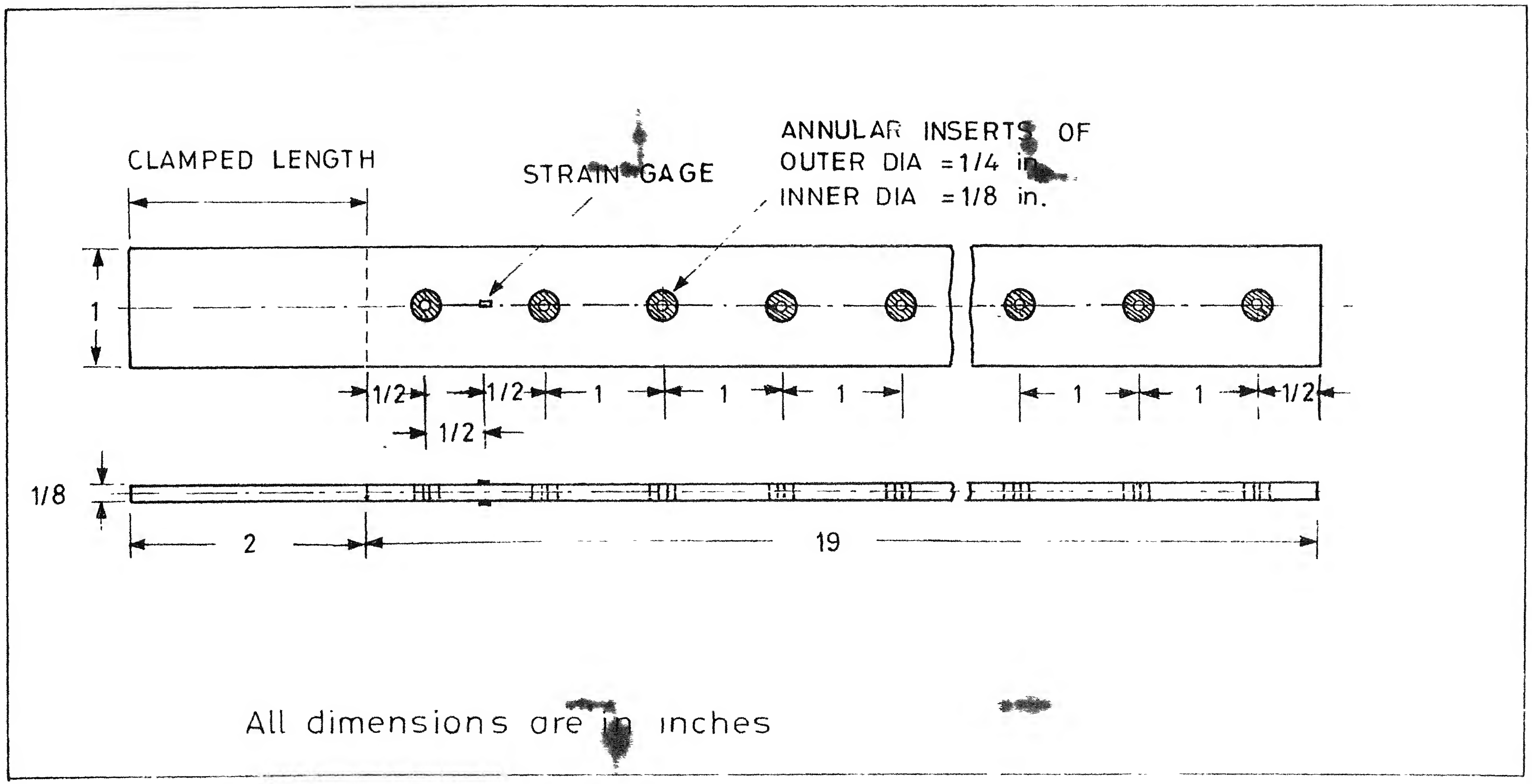


FIG. 3·3 DETAILS OF ALUMINIUM SPECIMEN WITH ANNULAR INSERTS

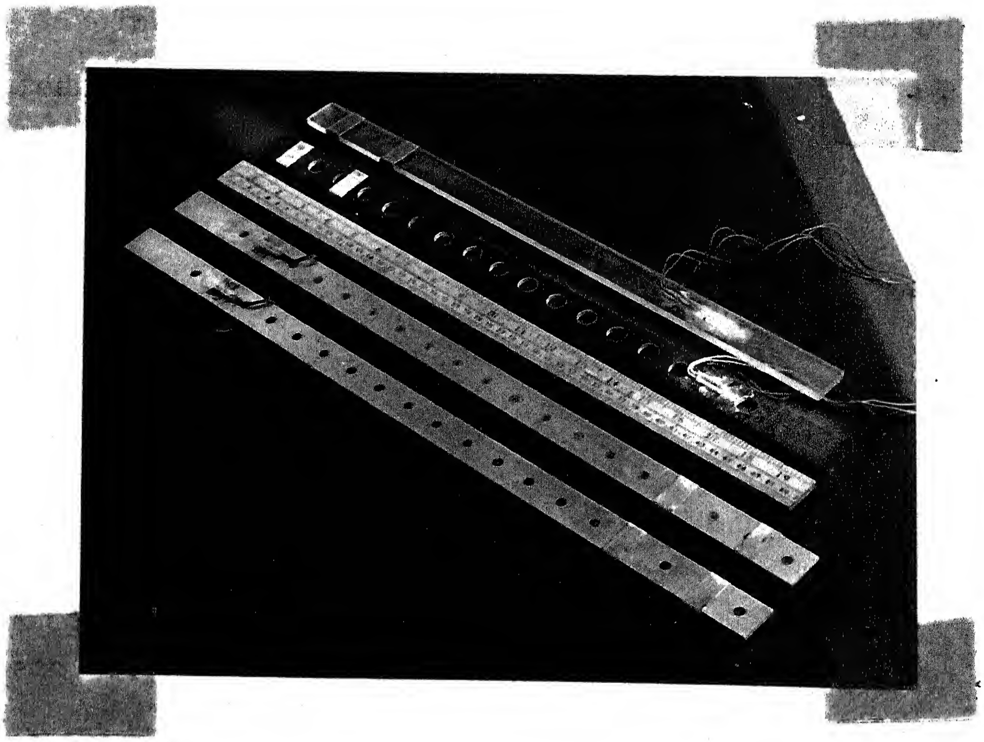


FIG. 3.4 DIFFERENT TYPES OF SPECIMENS

in a Budd Portable Strain Indicator. The output signal from the Strain Indicator was fed to a Honeywell Visicorder Oscillograph through a Krohn-Hite Band Pass Filter. The Band-Pass Filter was used to remove all parasitic frequencies coming along with the first natural frequency of the specimen.

The fundamental frequency of the specimen was measured by setting the specimen in free vibrations by means of the electro-magnet and feeding the output signal from the Strain Indicator directly to the Visicorder. The time period of the free oscillation was measured from the portion of this record after a lapse of a few initial cycles. This was done to ensure that the vibration is predominantly at the first mode as the higher modes are damped out much faster than this. Once this first natural frequency was known, the filter was tuned to pass this frequency. The plot obtained from the Visicorder was calibrated by giving a known strain in the Strain Indicator. This was done by bypassing the filter as the filter cannot pass a d.c signal. So the filter insertion loss factor at various frequencies was found out in another experiment and this was used in calculating the amplitude of strain from the record. Records of the specimens were taken at three different sensitivity settings of the Strain Indicator so as to facilitate ease and accuracy

of measurement at all levels of strain. A typical record of free vibration decay of a specimen is shown in Fig. 3.5.

Actually, only the upper half of the trace, as shown in Fig. 3.6, was recorded to increase the accuracy of the measurement. All specimens were tested at atmospheric pressure and in vacuum to study the effect of air damping.

The strain level during vibration was calculated by measuring the amplitude with the help of a divider and a diagonal scale and multiplying this value by the product of the calibration constant and the filter insertion loss factor. The logarithmic decrement at this strain level was calculated by measuring the next successive amplitude and using Eqn. (2.1.5). Equation (2.1.6) was employed wherever the difference between two successive amplitudes was not appreciable. A log-log graph of logarithmic decrement versus the amplitude of the measured strain was plotted from the above computations.

3.3 Computation of Material Damping Characteristics from Specimen Damping

Once the log-log plot of logarithmic decrement versus amplitude of measured strain is obtained, the material damping characteristics may be evaluated as outlined below.

FIG. 3.5 A TYPICAL FREE VIBRATION RECORD SHOWING THE FULL WAVE

FIG. 3.6 A TYPICAL RECORD FOR MEASUREMENT SHOWING THE HALF WAVE

For uniaxial stress systems, the relationship between the energy dissipation and the stress amplitude may be expressed as [5]

$$D_V = J\,\sigma^n \qquad (3.3.1)$$

where D_V = energy dissipated per unit volume of the material per cycle,

σ = harmonic stress amplitude,

J = damping constant, or the energy dissipated at unit stress amplitude, and

n = damping exponent.

J and n are material constants. The value of n may vary from 2 to 30. But most materials upto a stress level near the fatigue limit show a value between 2 and 3. The value of n = 2 indicates that only linear mechanisms contribute to overall damping capacity of the material. To cover a wide range of stress amplitudes, Eqn. (3.3.1) is expressed as [5]

$$D_V = \sum_i J_i \, \sigma^{n_i} \qquad (3.3.2)$$

Equation (3.3.1) yields the constant J with very peculiar units depending on the other constant, n. This peculiarity is removed if D_V is expressed in terms of the harmonic strain amplitude, ε , rather than that of the stress [12].

$$D_v = J' \varepsilon^n \qquad (3.3.3)$$

where J' = energy dissipated at unit strain amplitude

However, for most materials, to cover the low and the intermediate strain ranges, it is better to express D_v as follows [12]

$$D_v = J_1' \varepsilon^{n_1} + J_2' \varepsilon^{n_2} \qquad (3.3.4)$$

J_1', J_2', n_1 and n_2 represent the material damping characteristics. One of the major objectives of the thesis is to evaluate these constants for four different materials.

In the present work, the damping capacity of the specimen has been expressed in terms of an equivalent logarithmic decrement δ , given by Eqn. (2.1.7)

$$\delta = \frac{D_s}{2 W_s} \qquad (3.3.5)$$

The strain amplitudes have been taken to be the static strains under an equivalent static load P, acting at the free end of the specimen, which produces a tip deflection equal to the amplitude of oscillation at the free end. This assumption is justified as the cantilever specimen vibrates predominantly in the first mode [15].

Now, from simple bending theory, the strain (ε_x), in an element, at a distance x from the free end is given by

$$\varepsilon_x = \frac{P\ x\ y}{E\ I} \qquad (3.3.6)$$

where y = distance of the element from the neutral axis,

E = modulus of elasticity of the material of the specimen,

$I = \frac{b\ h^3}{12}$ = second moment of area of the cross section of the specimen about the neutral axis

with h = height of the specimen

and b = width of the specimen.

The energy dissipated by the specimen in one cycle can be obtained from Eqn. (3.3.3) as

$$D_s = 2\ J' \int_0^b \int_0^{h/2} \int_0^l \varepsilon_x^{\ n}\ dx\ dy\ dz \qquad (3.3.7)$$

Substituting for ε_x from Eqn. (3.3.6), we get

$$D_s = 2\ J' \int_0^b \int_0^{h/2} \int_0^l \left(\frac{P\ x\ y}{E\ I} \right)^n dx\ dy\ dz$$

Integration over the limits yields

$$D_s = 2\ b\ J' \ \frac{P^n}{(E\ I)^n} \ \frac{(\frac{h}{2})^{n+1} \ l^{n+1}}{(n+1)^2} \tag{3.3.8}$$

Applying Eqn. (3.3.6) to the amplitude of measured strain ε_m, we have

$$\varepsilon_m = \frac{P\ l_1 \frac{h}{2}}{E\ I} \tag{3.3.9}$$

where l_1 = distance of the centre of the strain gage from the free end of the cantilever.

Using Eqns. (3.3.8) and (3.3.9) we finally obtain the relationship between D_s and ε_m as

$$D_s = \frac{J'}{(n+1)^2} \left(\frac{l}{l_1} \right)^n (\varepsilon_m)^n \ V \tag{3.3.10}$$

where $V = b\ h\ l$ = volume of the specimen.

The maximum elastic energy stored in the specimen during a cycle, W_s, is given by the expression

$$W_s = \frac{1}{2} P\Delta \tag{3.3.11}$$

where Δ = tip deflection of the specimen with an end load P.

For a cantilever, Δ is given by

$$\Delta = \frac{P\ l^3}{3\ E\ I} \tag{3.3.12}$$

Substitution of Eqn. (3.3.12) in Eqn. (3.3.11) yields

$$W_s = \frac{1}{2} \frac{P^2 l^3}{3 E I} \tag{3.3.13}$$

Using Eqn. (3.3.9) in Eqn. (3.3.13) we get

$$W_s = \frac{2}{3} \frac{l^3 E I}{l_1^2 h^2} \varepsilon_m^2 \tag{3.3.14}$$

By substituting Eqns. (3.3.10) and (3.3.14) in Eqn. (3.3.5), we arrive at the final expression for δ as,

$$\delta = \frac{3}{4} \frac{V h^2}{E I l} \frac{J'}{(n+1)^2} \left(\frac{l}{l_1} \right)^{n-2} \varepsilon_m^{n-2} \tag{3.3.15}$$

Further simplification of the above expression using $I = \frac{b h^3}{12}$ and $V = b h l$ gives,

$$\delta = \frac{9}{E} \frac{J'}{(n+1)^2} \left(\frac{l}{l_1} \right)^{n-2} \varepsilon_m^{n-2} \tag{3.3.16}$$

Equation (3.3.16) can be written as,

$$\delta = K \varepsilon_m^{n-2} \tag{3.3.17}$$

where $$K = \frac{9}{E} \frac{J'}{(n+1)^2} \left(\frac{l}{l_1} \right)^{n-2} \tag{3.3.18}$$

From Eqn. (3.3.17) it can be seen that the slope of the log-log plot of δ vs ε_m will be equal to $n - 2$. Hence the value of n is obtained from the plot. Using this value of n, K is obtained from Eqn. (3.3.17), and finally J' is evaluated from Eqn. (3.3.18).

As already mentioned earlier, if the material damping characteristics is to be expressed by Eqn. (3.3.4), the corresponding expression for the logarithmic decrement of the specimen can easily be seen to be of the form,

$$\delta = K_1 \varepsilon_m^{n_1-2} + K_2 \varepsilon_m^{n_2-2} \quad \text{from Eqn. (3.3.17)} \qquad (3.3.19)$$

$$\left.\begin{aligned} \text{with} \quad K_1 &= \frac{9}{E} \frac{J_1'}{(n_1 + 1)^2} \left(\frac{1}{l_1}\right)^{n_1-2} \\ \text{and} \quad K_2 &= \frac{9}{E} \frac{J_2'}{(n_2 + 1)^2} \left(\frac{1}{l_1}\right)^{n_2-2} \end{aligned}\right\} \text{from Eqn. (3.3.18)} \qquad (3.3.20)$$

Now, from the best-fit curves drawn through the experimental points of δ vs ε_m, values of n_1, n_2, K_1 and K_2 are deduced by using Eqn. (3.3.19). J_1' and J_2' are evaluated from the values of K_1 and K_2 respectively, by using Eqns. (3.3.20). The average slopes of the log-log plot of δ vs ε_m at the lower and higher levels of strain are equal to $(n_1 - 2)$ and $(n_2 - 2)$ respectively.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Determination of Proper Clamping Torque

First, in order to ascertain the necessary clamping force to minimize slip-damping at the support, free vibration records for the Aluminium specimen were obtained with various values of the clamping torque. The values of δ evaluated at a certain ε_m for all these cases are shown in Fig. 4.1. It can be observed that a clamping torque more than 40 ft lbs is sufficient to ensure a proper clamped end condition. Throughout the experiments, a clamping torque of 50 ft. lbs. was maintained with the help of a torque wrench.

4.2 Evaluation of Damping Constants of Materials

Four different materials, namely, Aluminium, Cast Iron, Bakelite and Perspex were tested and the damping constants of these materials were computed by the method outlined in Chapter 3.

(i) Aluminium: For calculating the logarithmic decrement δ of the Aluminium specimen, Eqn. (2.1.6) was used with the following values of p for different strain ranges.

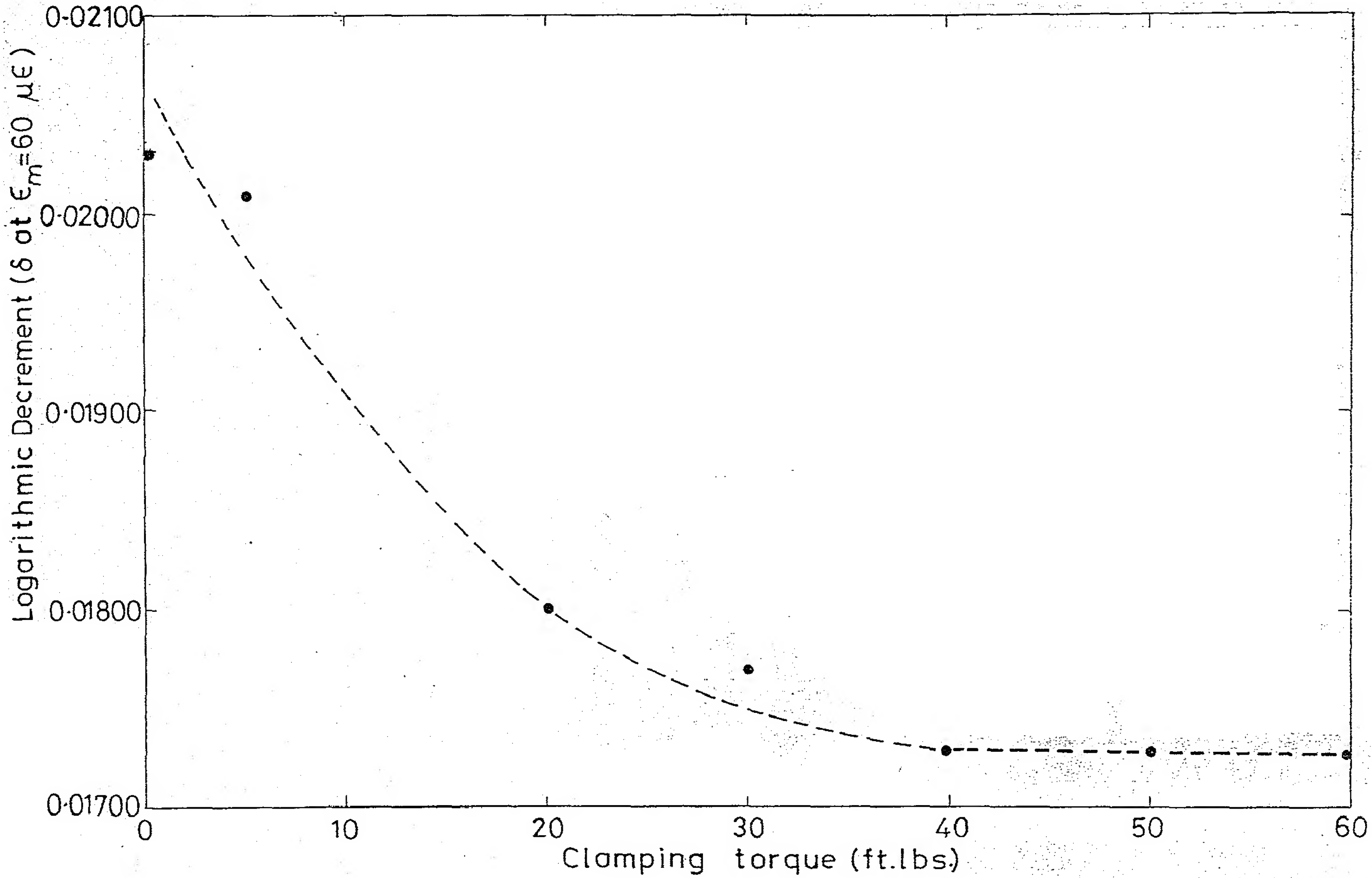


FIG 4·1 VARIATION OF OVERALL DAMPING OF ALUMINIUM SPECIMEN WITH CLAMPING TORQUE

$p = 1$ for $\varepsilon_m > 80\ \mu\varepsilon$

$p = 4$ for $80\ \mu\varepsilon > \varepsilon_m > 1\ \mu\varepsilon$

$p = 8$ for $1\ \mu\varepsilon > \varepsilon_m > 0.5\ \mu\varepsilon$

and $p = 16$ for $0.5\ \mu\varepsilon > \varepsilon_m > 0\ \mu\varepsilon$

where ε_m is the amplitude of the measured strain.

The above values of p were chosen to increase the accuracy of calculations. The use of a value for p more than unity is justified from the fact that the value of δ does not change significantly with strain amplitude in these ranges of ε_m.

Log-log plots of δ vs ε_m for three different air pressures are shown in Fig. 4.2. It can be observed from the plots that for such low damping materials, air damping plays an important role above a certain value of the amplitude (30 $\mu\varepsilon$ approximately, in this case) even at such a low frequency (10.75 Hz). The damping due to the air, being an increasing function of the velocity of oscillation is seen to increase as the amplitude of vibration increases. It can also be seen that the overall damping of the specimen, (even at the maximum strain level considered in this work), measured at a pressure of 200 mm Hg is not significantly higher than that at a pressure of 25 mm Hg. Hence it was concluded that even for the Aluminium specimens, the air damping can be neglected if

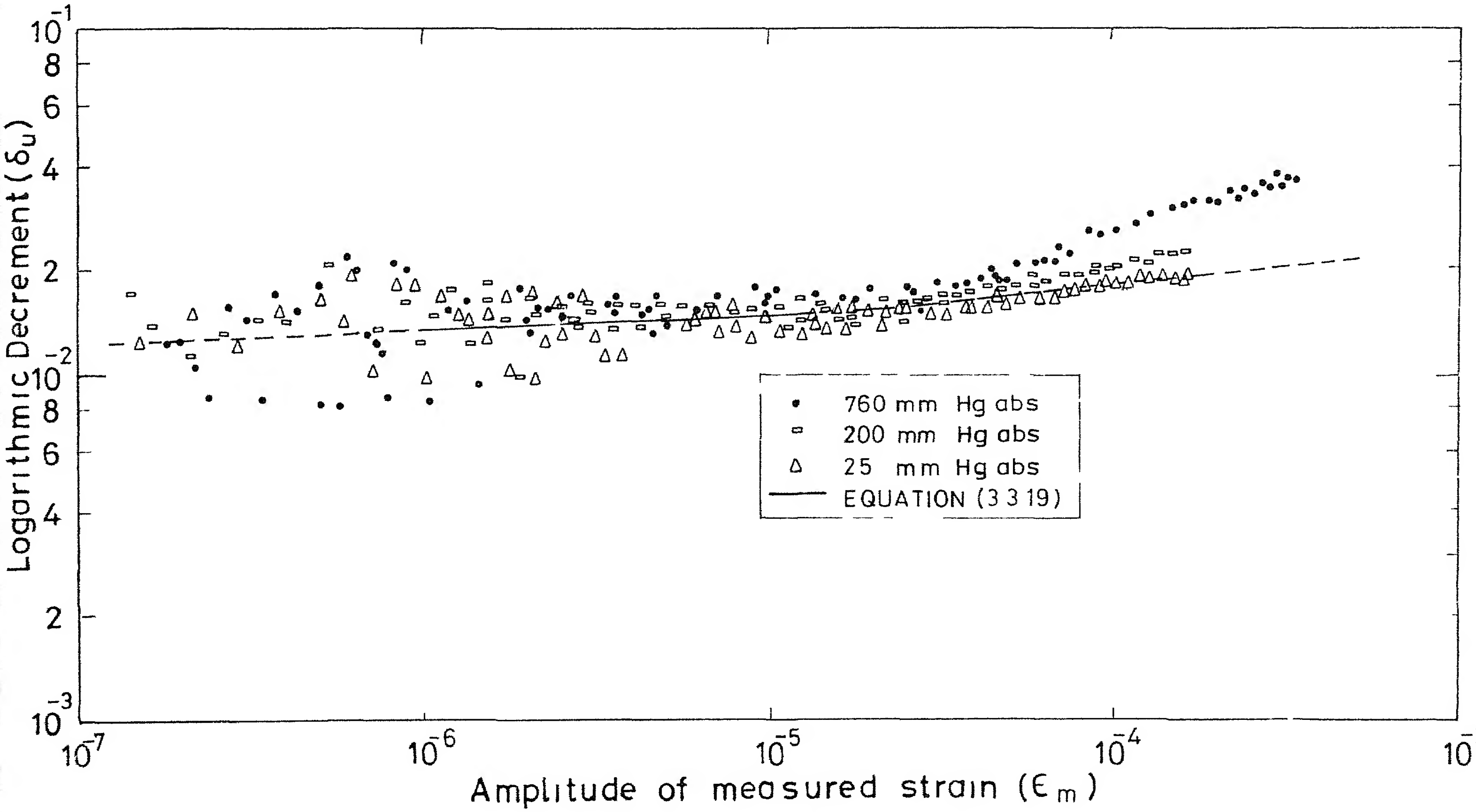


FIG. 4 2 DAMPING OF ALUMINIUM SPECIMEN FOR VARIOUS VALUES OF AIR PRESSURE

the experiments are conducted at a pressure of 25 mm Hg. All subsequent experiments on Aluminium specimens have been carried out at 25 mm Hg pressure.

The large scatter in the results at low levels of strain can be attributed to the following reasons:

(a) loss in the accuracy of measurement of the strain amplitudes as the diagonal scale reads these correctly upto two decimal places only,

(b) extraneous losses, e.g., losses at the clamp, may be comparable to the material damping being measured,

(c) the sensitivity of the strain gages and that of the strain indicator are not reliable.

Below a certain value of ε_m, the value of δ remains constant. This indicates that only linear damping mechanisms are significant at such low levels of strain.

(ii) Cast Iron: The specimens used were made from Grey Cast Iron of the following composition:

Carbon	3.3%
Silicon	1.93%
Sulphur	0.047%

Values of p used in Eqn. (2.1.6) to evaluate δ were

$p = 1$ for $\varepsilon_m > 5\,\mu\varepsilon$

$p = 4$ for $5\,\mu\varepsilon > \varepsilon_m > 0.5\,\mu\varepsilon$

and $p = 8$ for $0.5\,\mu\varepsilon > \varepsilon_m > 0\,\mu\varepsilon$

It can be noticed here that $p = 1$ has been used for a lesser value of ε_m than that in the case of Aluminium. This is because Cast Iron has a much higher damping capacity than Aluminium. In the record of a Cast Iron specimen, the difference in amplitudes between two successive peaks is quite significant and so the accuracy in computation of δ is not reduced by using lower values for p. For such high damping materials, scatter in the results is also considerably reduced.

As can be observed from the log-log plot of δ vs ε_m, shown in Fig. 4.3, air damping for the Cast Iron specimen is negligible even at high amplitudes. So all subsequent measurements on Cast Iron specimens have been conducted at atmospheric pressure.

The Cast Iron specimen behaves linear at low levels of strain as δ remains constant.

(iii) Bakelite: Values of p used in Eqn. (2.1.6) to evaluate δ were

$$p = 1 \quad \text{for} \quad \varepsilon_m > 2\,\mu\varepsilon$$

$$\text{and} \quad p = 4 \quad \text{for} \quad 2\mu\varepsilon > \varepsilon_m > 0\,\mu\varepsilon$$

From Fig. 4.4 it can be seen that $p = 4$ has been used only in the region where the Bakelite specimen behaves linear (i.e. δ = constant). This is the range where δ is independent of the value of p.

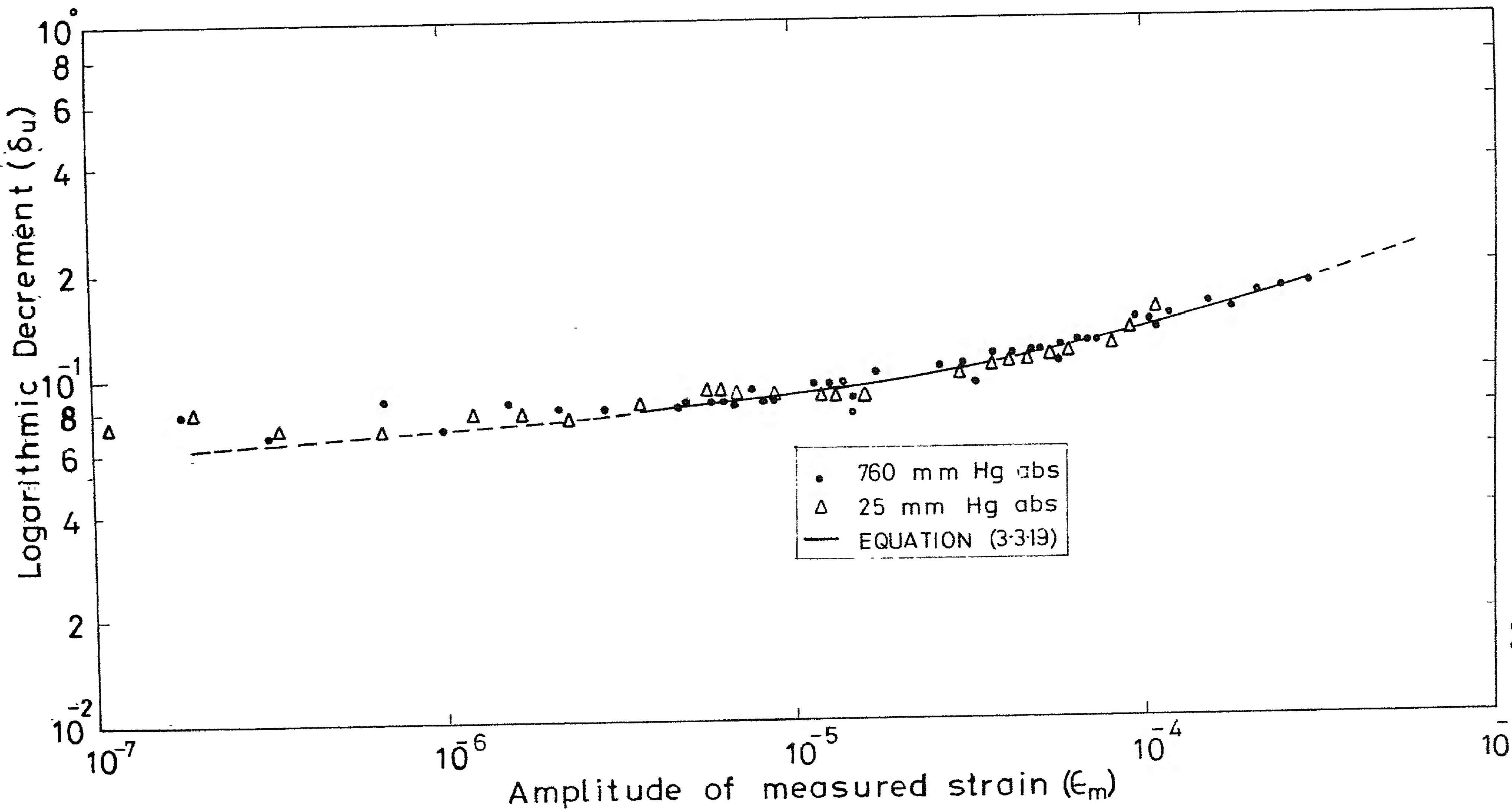


FIG. 4·3 DAMPING OF CAST IRON SPECIMEN AT DIFFERENT LEVELS OF STRAIN

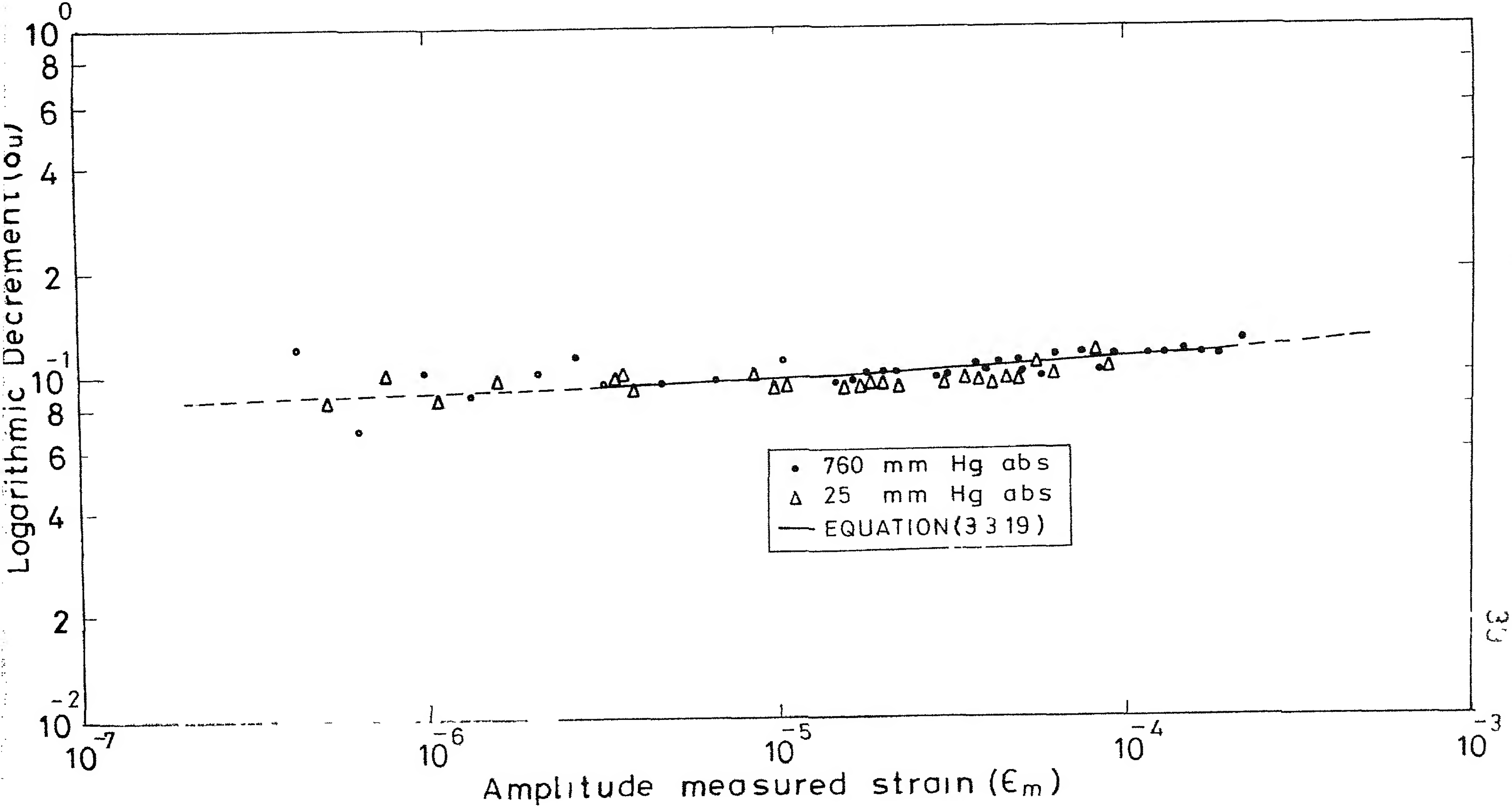


FIG 4 4 DAMPING OF BAKELITE SPECIMEN AT DIFFERENT LEVELS OF STRAIN

Since Bakelite is also a high damping material, air damping is seen to be negligible and scatter is also small.

(iv) Perspex: In the case of the Perspex specimen, $p = 1$ was used at all levels of strain in the calculation of δ from Eqn. (2.1.6). This was possible due to the high damping of the Perspex specimen as compared to the previous ones. This is partly due to the high damping of the material itself and partly due to the larger size of the specimen.

As was the case with Cast Iron and Bakelite specimens, air damping is negligible for the Perspex specimen and scatter is also small.

Figure 4.5 shows that δ is constant for the entire range of ϵ_m. This indicates that Perspex behaves as a linear material in this strain range.

Best-fit curves were drawn in Figs. 4.2 to 4.5 and the values of n_1, n_2, K_1, K_2, J_1' and J_2' were deduced in each case using Eqns. (3.3.19) and (3.3.20). These values are given in Table 4.1.

It was noted that for Aluminium, the damping capacity obtained (in the linear range) in the present work agrees very well with that predicted by the thermoelastic theory of Zener [16].

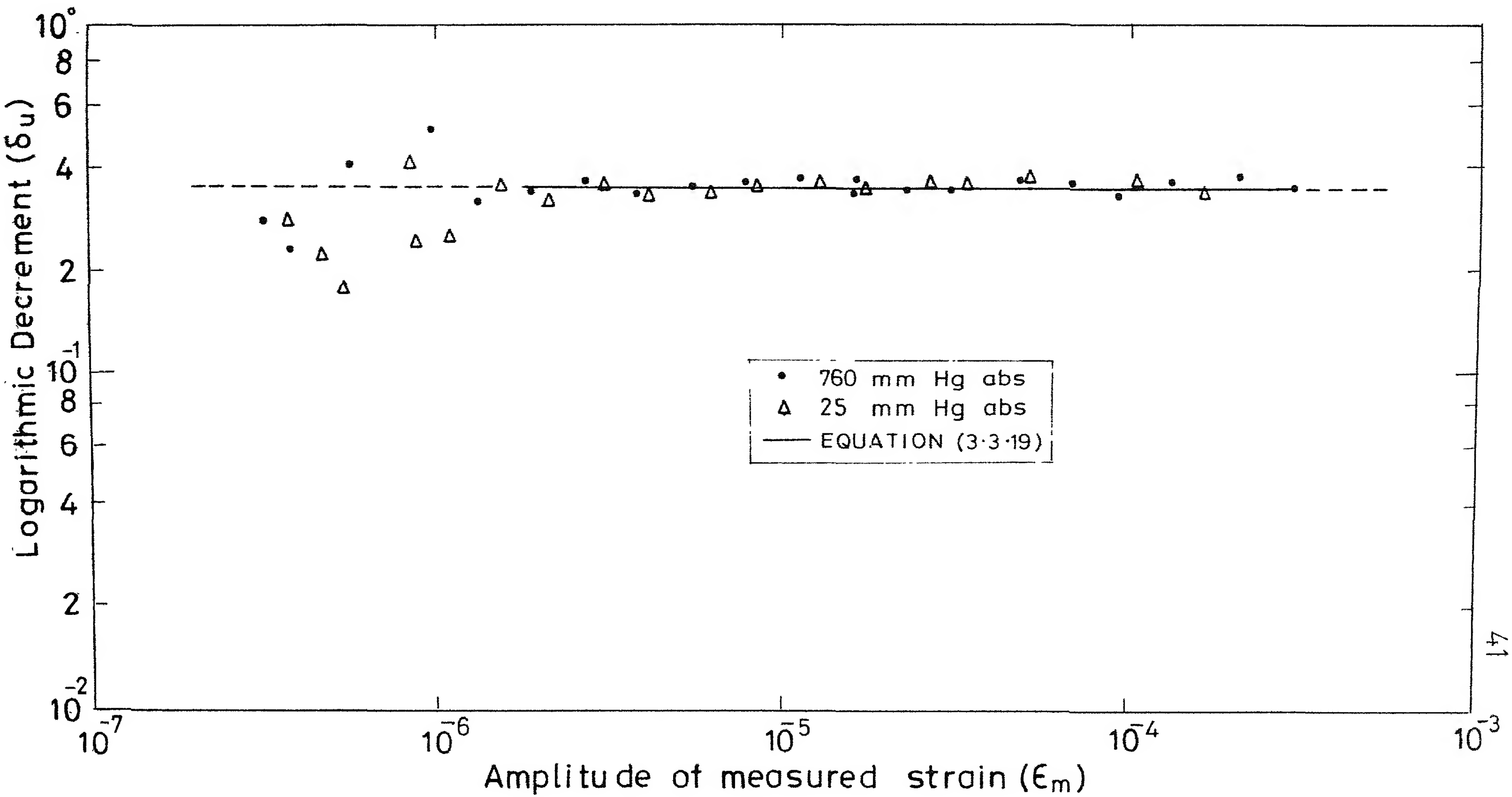


FIG. 4·5 DAMPING OF PERSPEX SPECIMEN AT DIFFERENT LEVELS OF STRAIN

TABLE 4.1

MATERIAL PROPERTIES DETERMINED FROM THE EXPERIMENTS

MATERIAL	K_1	K_2	J'_1 (lbf/sq.in/cycle)	J'_2 (lbf/sq.in/cycle)	n_1	n_2
Aluminium	8.1537×10^{-3}	4.0494×10^{-2}	0.8561×10^{5}	4.6387×10^{5}	2.0000	2.1459
Cast Iron	5.7221×10^{-2}	3.3597	8.5833×10^{5}	629.3778×10^{5}	2.0000	2.4040
Bakelite	7.8059×10^{-2}	2.8694×10^{-1}	3.1224×10^{5}	13.0996×10^{5}	2.0000	2.2309
Perspex	3.6710×10^{-1}	-	1.5835×10^{5}	-	2.0000	-

It should be noted that the quantity $(n_2 - 2)$ indicates the magnitude of non-linearity in the damping mechanisms. Moreover, the ratio J_2'/J_1' indicates the level of strain at which the non-linearity in the damping mechanisms commences. Higher value of the ratio signifies commencement of non-linearity at lower strain levels. From the values presented in Table 4.1, it can easily be seen that for Cast Iron, non-linear mechanisms are predominant even at low values of strain. Also the extent of non-linearity is more than that in other materials. This is believed to be due to the contribution from magneto-elastic damping which is a highly non-linear mechanism with a damping index $n = 3$.

4.3 Effect of Introduced Stress Concentration on Specimen Damping

Stress concentration in the Cast Iron and Bakelite specimens was introduced by drilling holes as already explained in Chapter 3. A non-dimensional quantity, λ, has been used to specify the size of the holes, where

$$\lambda = \frac{\text{diameter of the hole}}{\text{width of the specimen}}$$

Both the specimens were tested with three different values of λ, $\lambda = 0.25, 0.4$ and 0.5. Log-log plots of δ vs ϵ_m for these cases are shown in Figs. 4.6 to 4.11.

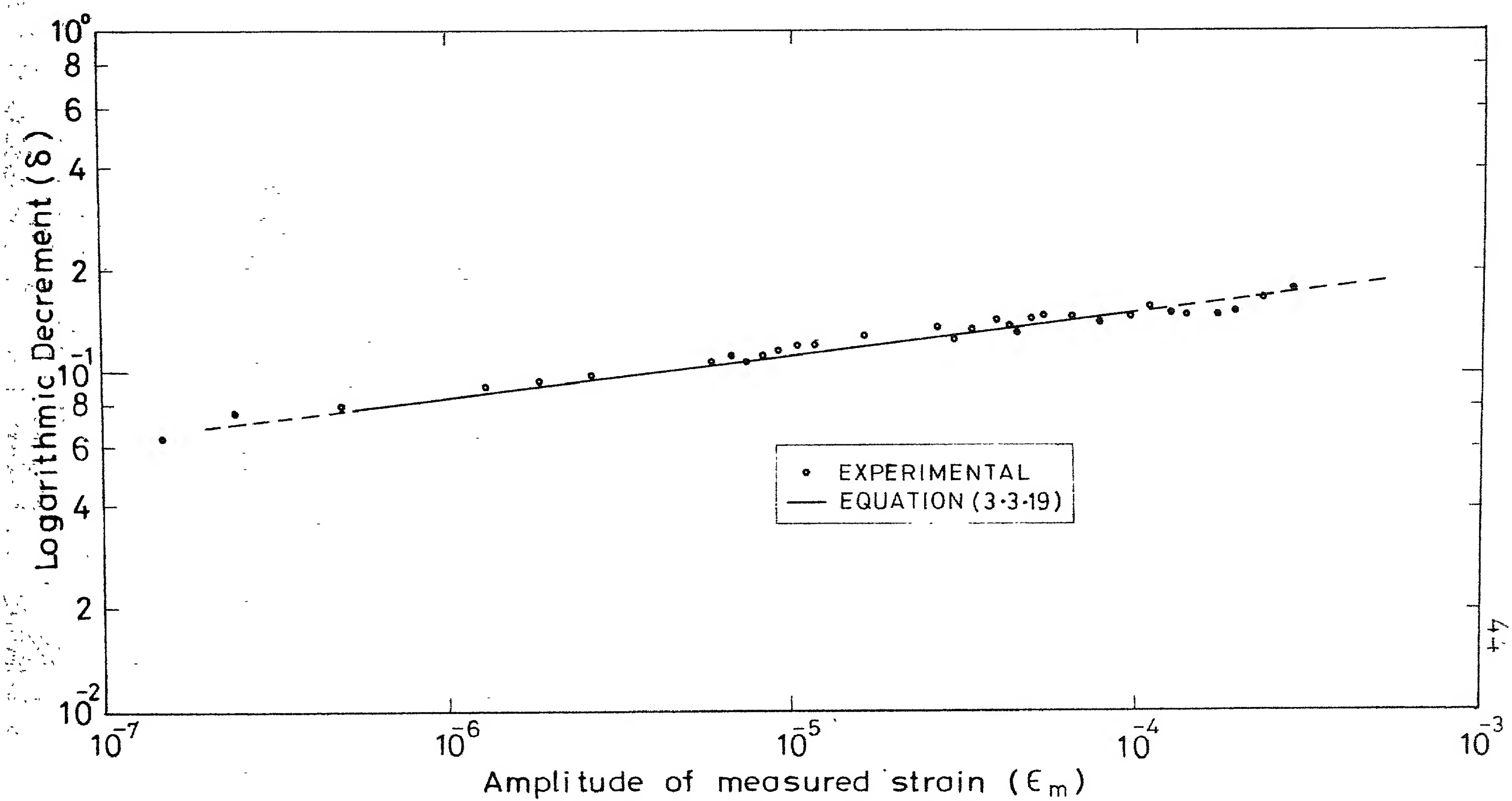


FIG.4·6 DAMPING OF CAST IRON SPECIMEN WITH HOLE SIZE $\lambda = 0{\cdot}25$

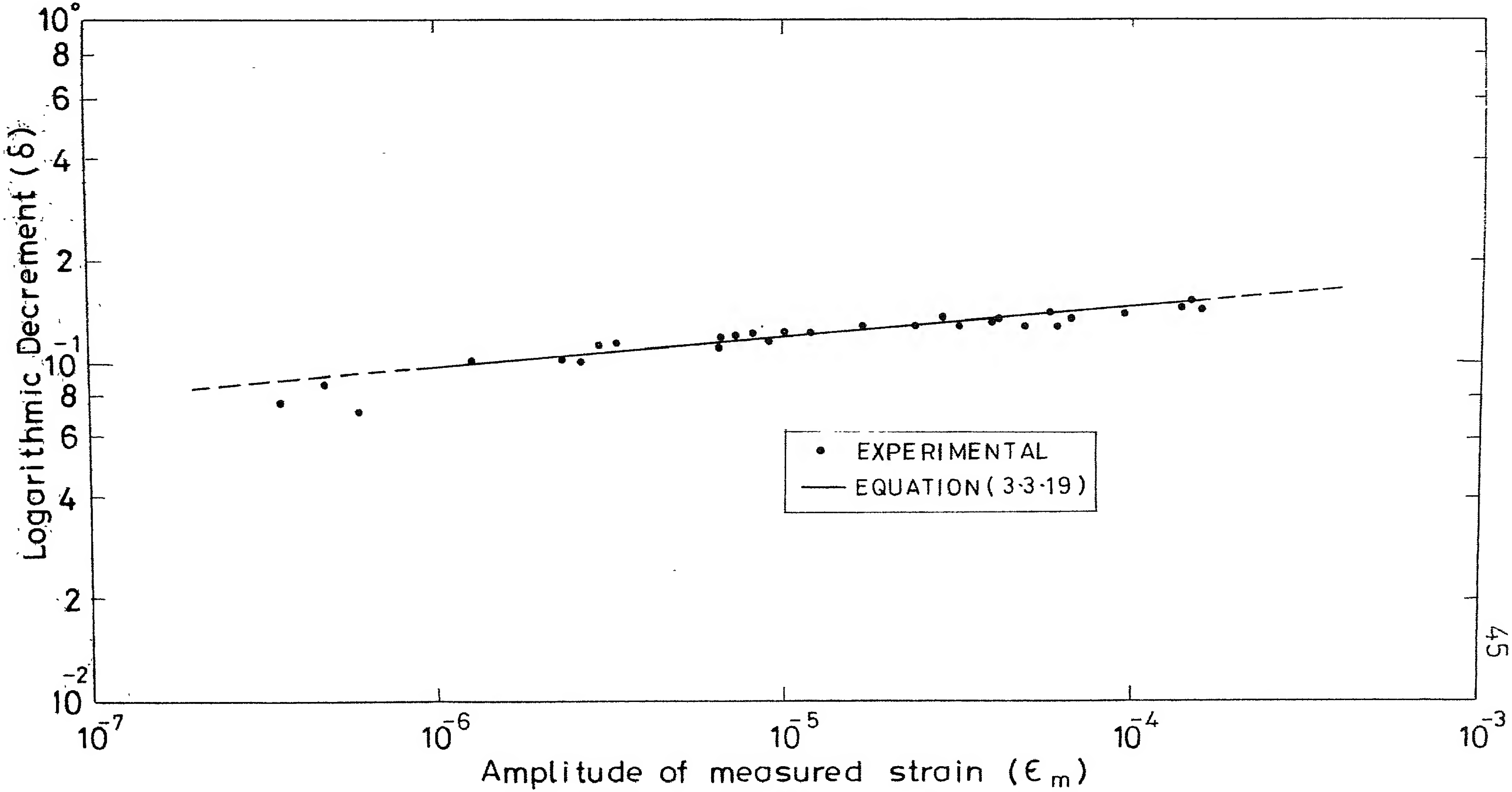


FIG.4·7 DAMPING OF CAST IRON SPECIMEN WITH HOLE SIZE $\lambda = 0{\cdot}4$

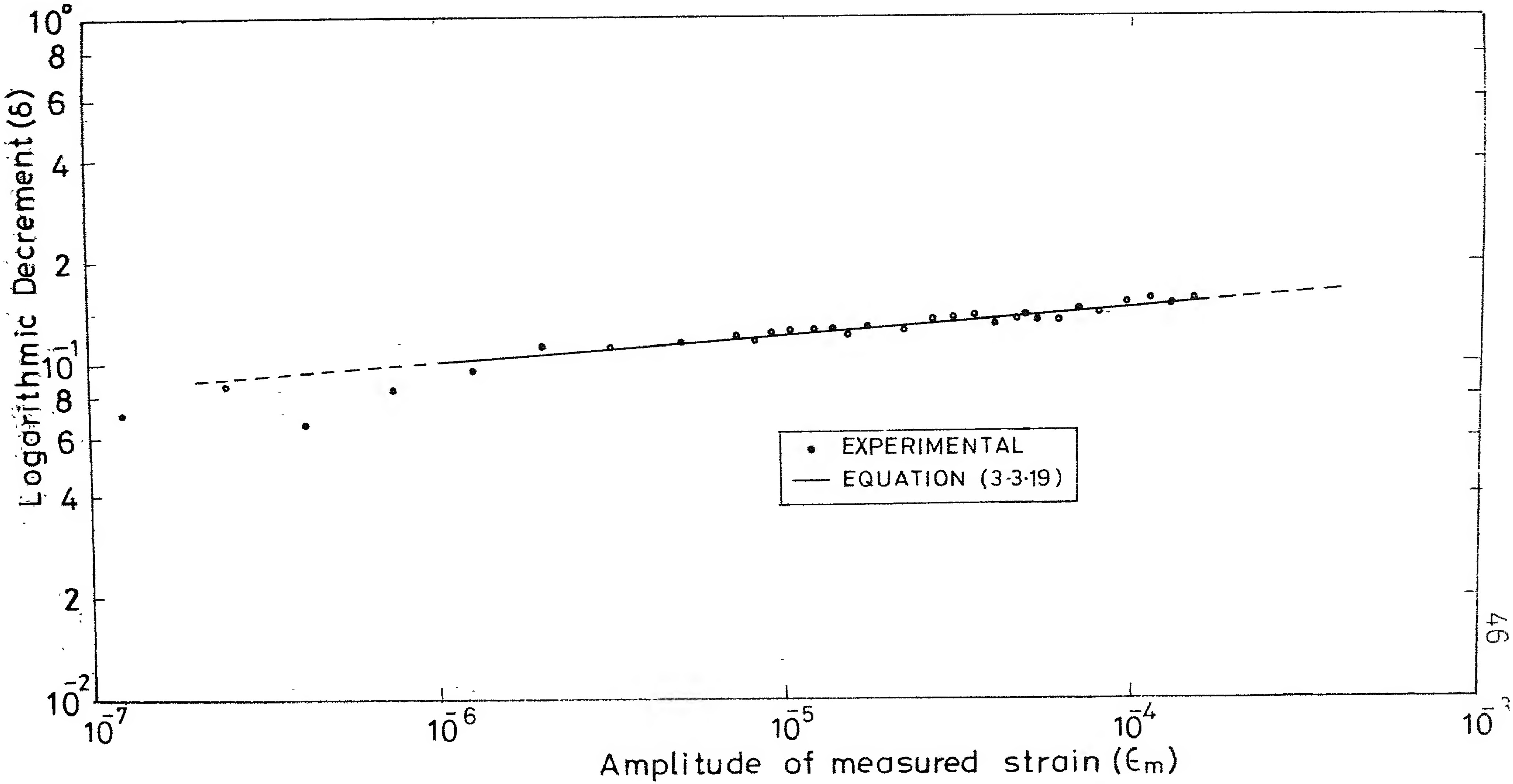


FIG. 4·8 DAMPING OF CAST IRON SPECIMEN WITH HOLE SIZE $\lambda = 0 \cdot 5$

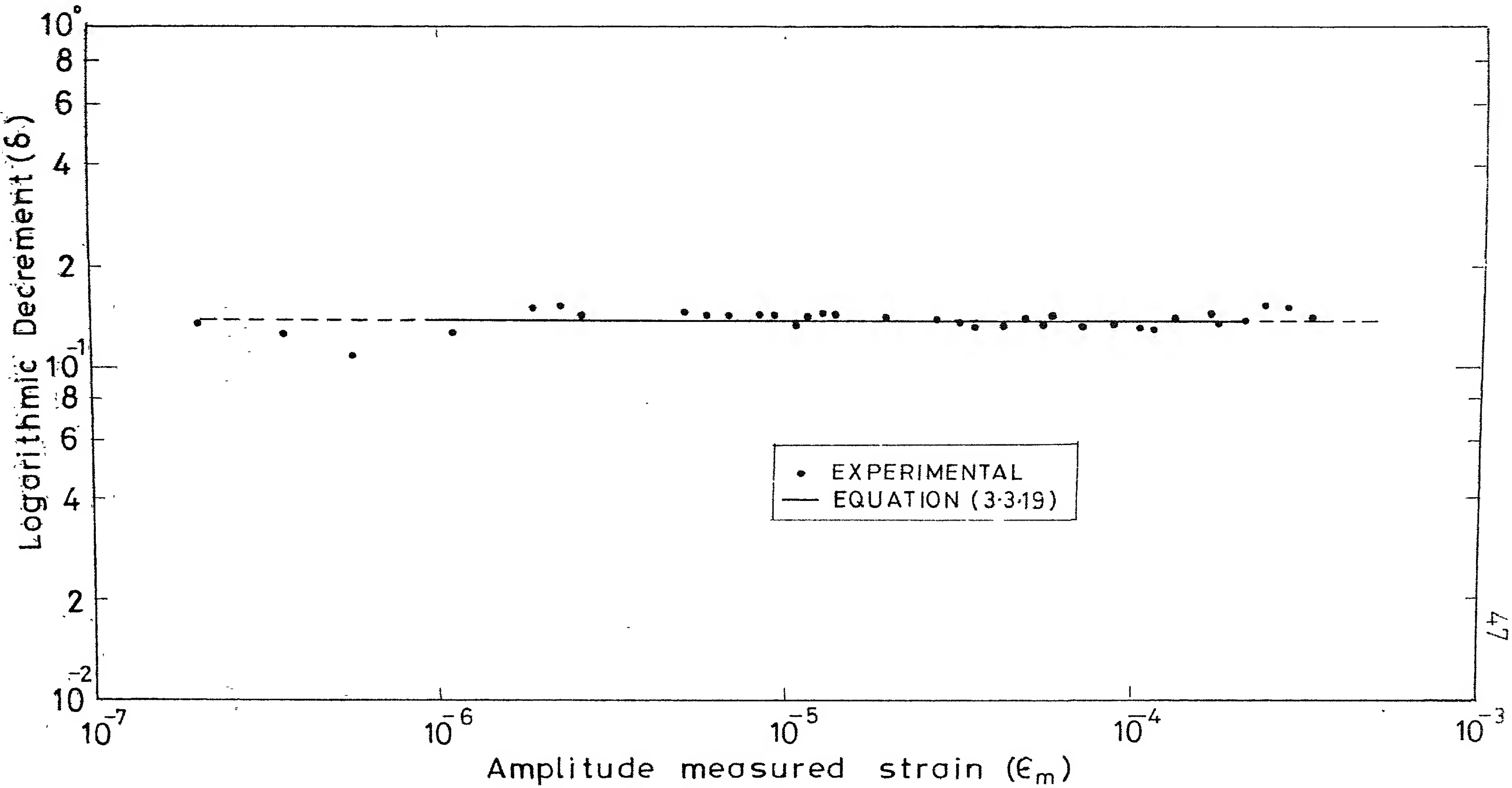


FIG.4·9 DAMPING OF BAKELITE SPECIMEN WITH HOLE SIZE $\lambda = 0{\cdot}25$

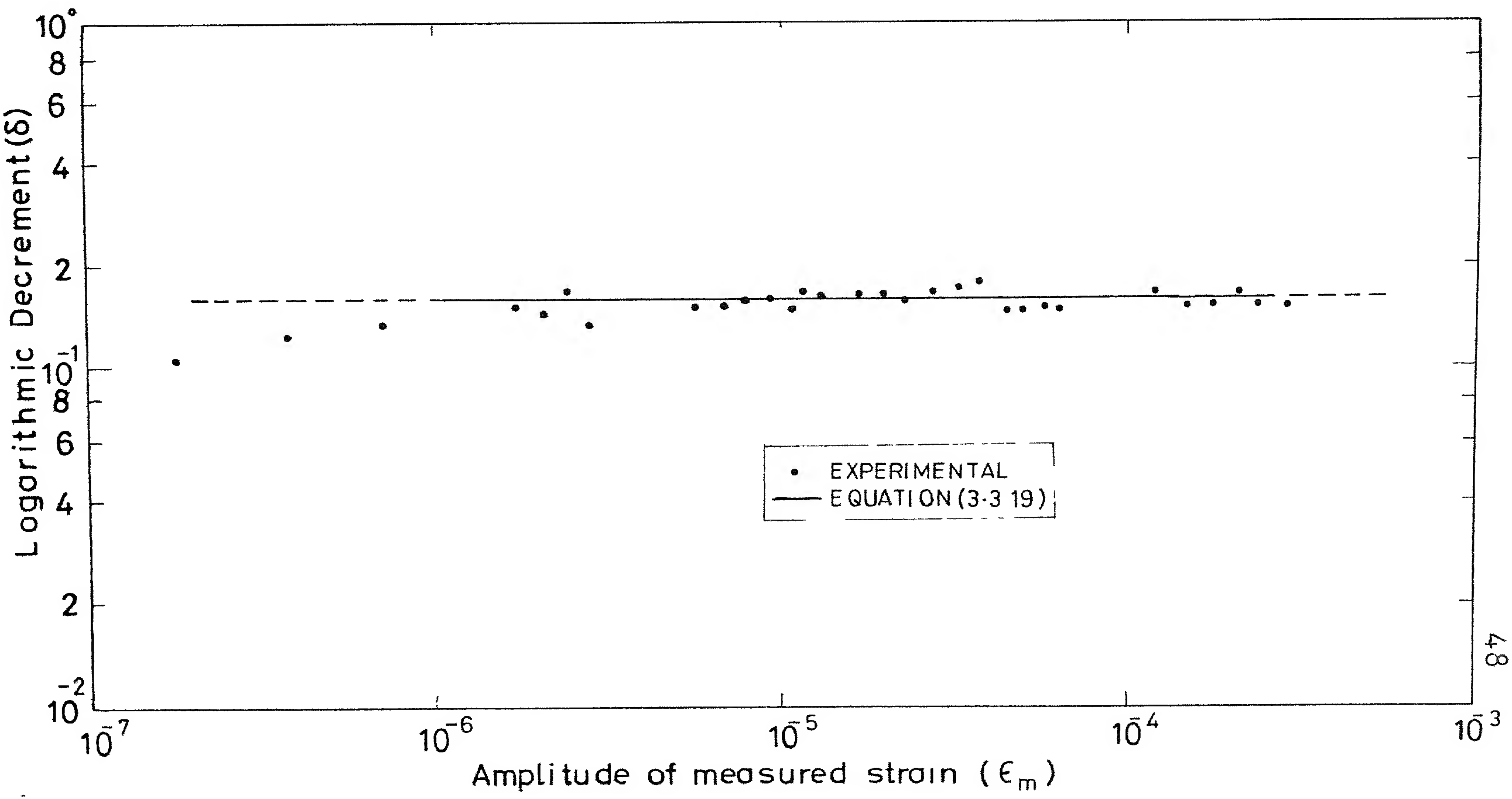


FIG 4·10 DAMPING OF BAKELITE SPECIMEN WITH HOLE SIZE $\lambda = 0{\cdot}4$

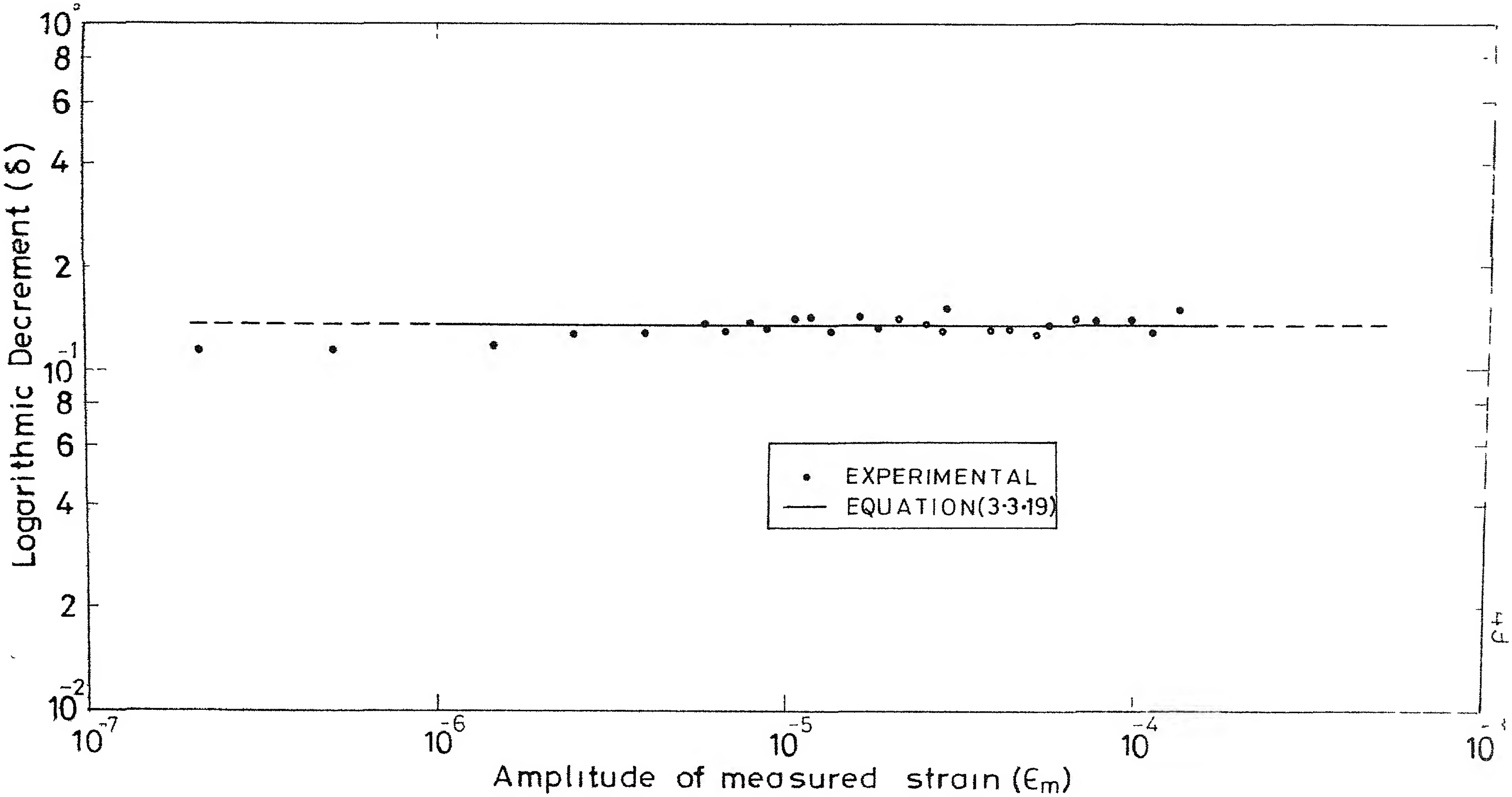


FIG 4·11 DAMPING OF BAKELITE SPECIMEN WITH HOLE SIZE $\lambda = 0{\cdot}5$

The increment in the damping capacity is expressed as δ / δ_u , where δ_u refers to the logarithmic decrement of the undisturbed (hole-free) specimen for the same amplitude of measured strain. The values of δ and δ_u have been taken from the best-fit curves to the experimental data. Results obtained for the variation in damping capacities of the Cast Iron and Bakelite specimens are presented in Figs. 4.12 and 4.13.

It can be observed that for the Cast Iron specimen, improvement in damping capacity with holes is predominant only at low levels of measured strain (ϵ_m). At higher levels of ϵ_m, there is a decrement in the damping capacity of the specimen with holes.

The average increment in the overall damping of the specimen with holes is observed to be negligible. This result is also expected from the theoretical analysis presented in reference [6], because of the low value of the index n_2 obtained for Cast Iron.

In the case of the Bakelite specimen (Fig. 4.13) there is a marked improvement in the damping capacity at all levels of measured strain (ϵ_m). Moreover, there is an optimum size of hole for which the improvement is maximum. The existence of this optimum size was shown in the theoretical analysis also [6]. This is obviously the nett outcome of the contradictory effects of stress

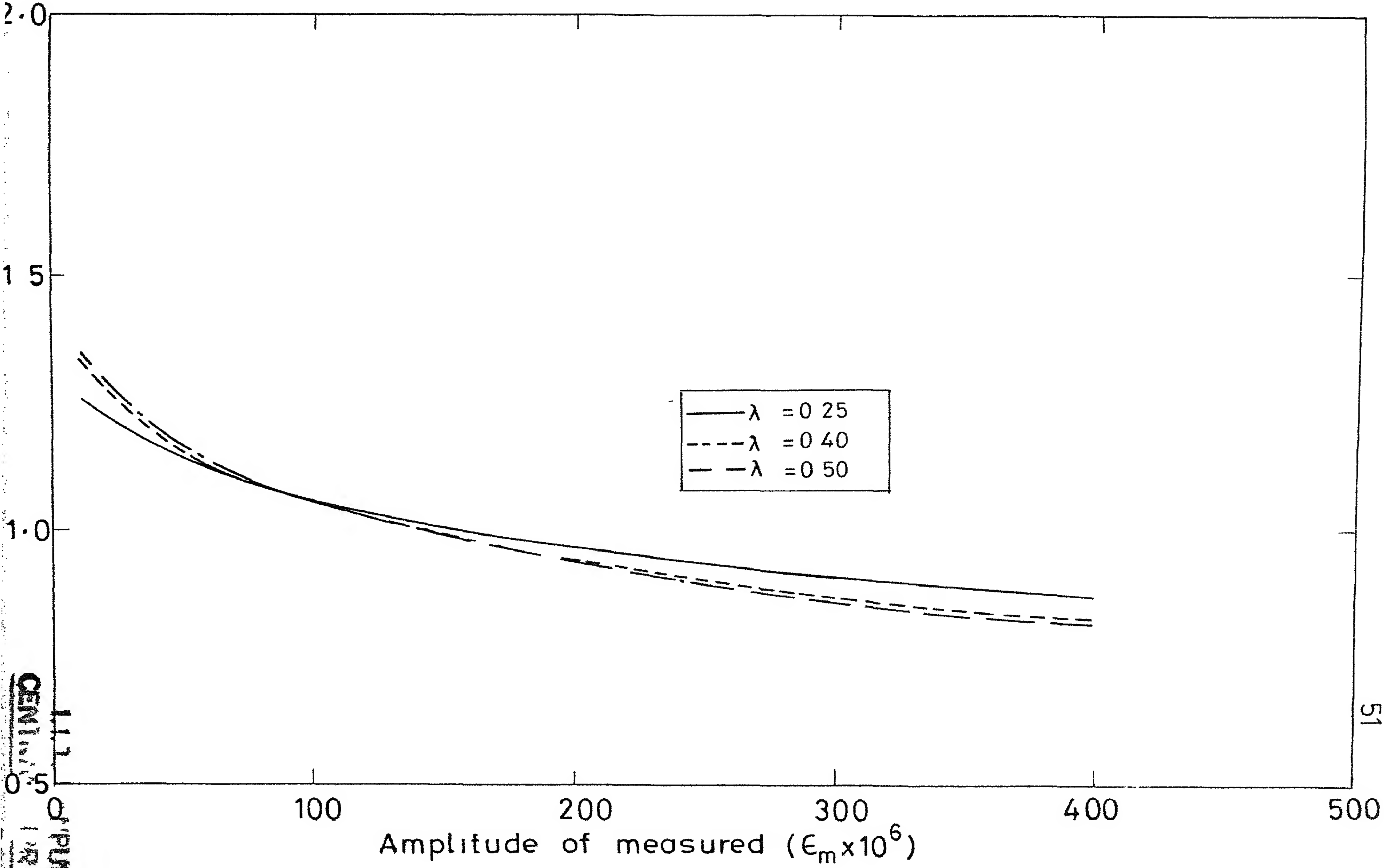


FIG 4 12 VARIATION OF DAMPING CAPACITY OF CAST IRON SPECIMEN WITH DIFFERENT HOLE SIZES

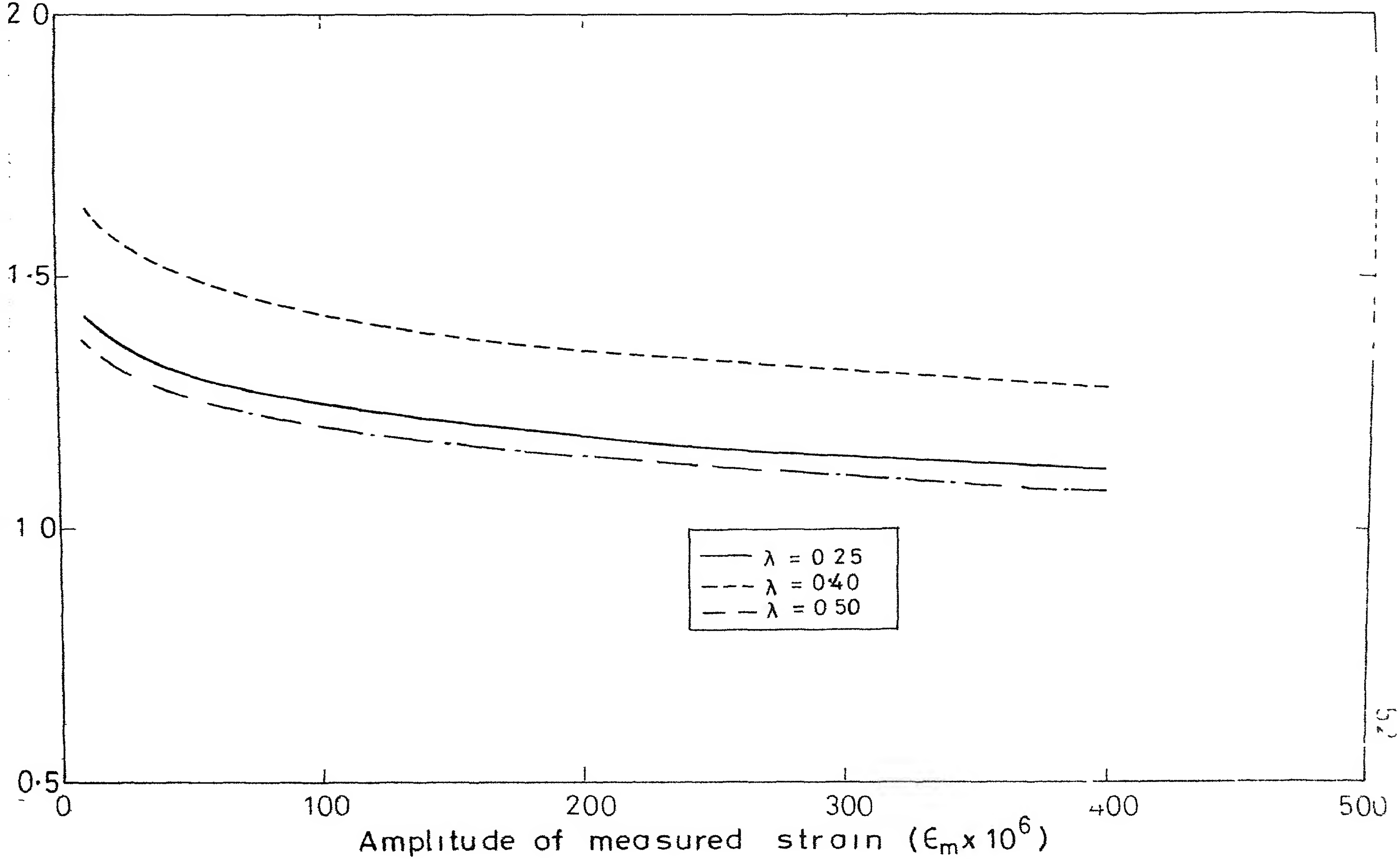


FIG.4 13 IMPROVEMENT OF DAMPING CAPACITY OF BAKELITE SPECIMEN WITH DIFFERENT HOLE SIZES

concentration and loss of material. The optimum size of the hole i.e. $\lambda = 0.4$, is also the same as that predicted by the theory. However, it should be emphasised that Bakelite shows a marked departure from linear elastic behaviour which was assumed in the theoretical analysis.

The average increment in the overall damping of the Bakelite specimen is seen to be of the order of 25% with $\lambda = 0.25$, 45% with $\lambda = 0.4$ and 20% with $\lambda = 0.5$.

To assess the loss in the static rigidity of the specimens due to the holes, a simple experiment was conducted. The specimens were clamped in the set up and the free end was loaded using a pan with weights. The corresponding deflection at the tip was measured using a dial gauge. These results are presented in Figs. 4.14 and 4.15. The static rigidities of the specimens have been obtained from the slopes of these force vs deflection curves. The loss in static rigidity has been expressed as K/K_u, where K and K_u are the stiffnesses of the specimen with and without holes respectively. The variation of this non-dimensional quantity with the size of the holes is shown in Fig. 4.16.

It can be seen that for the Bakelite specimen, with optimum sized holes ($\lambda = 0.4$), an average increment in the damping capacity of the order of 45% can be achieved at the expense of only 6% loss in the static rigidity.

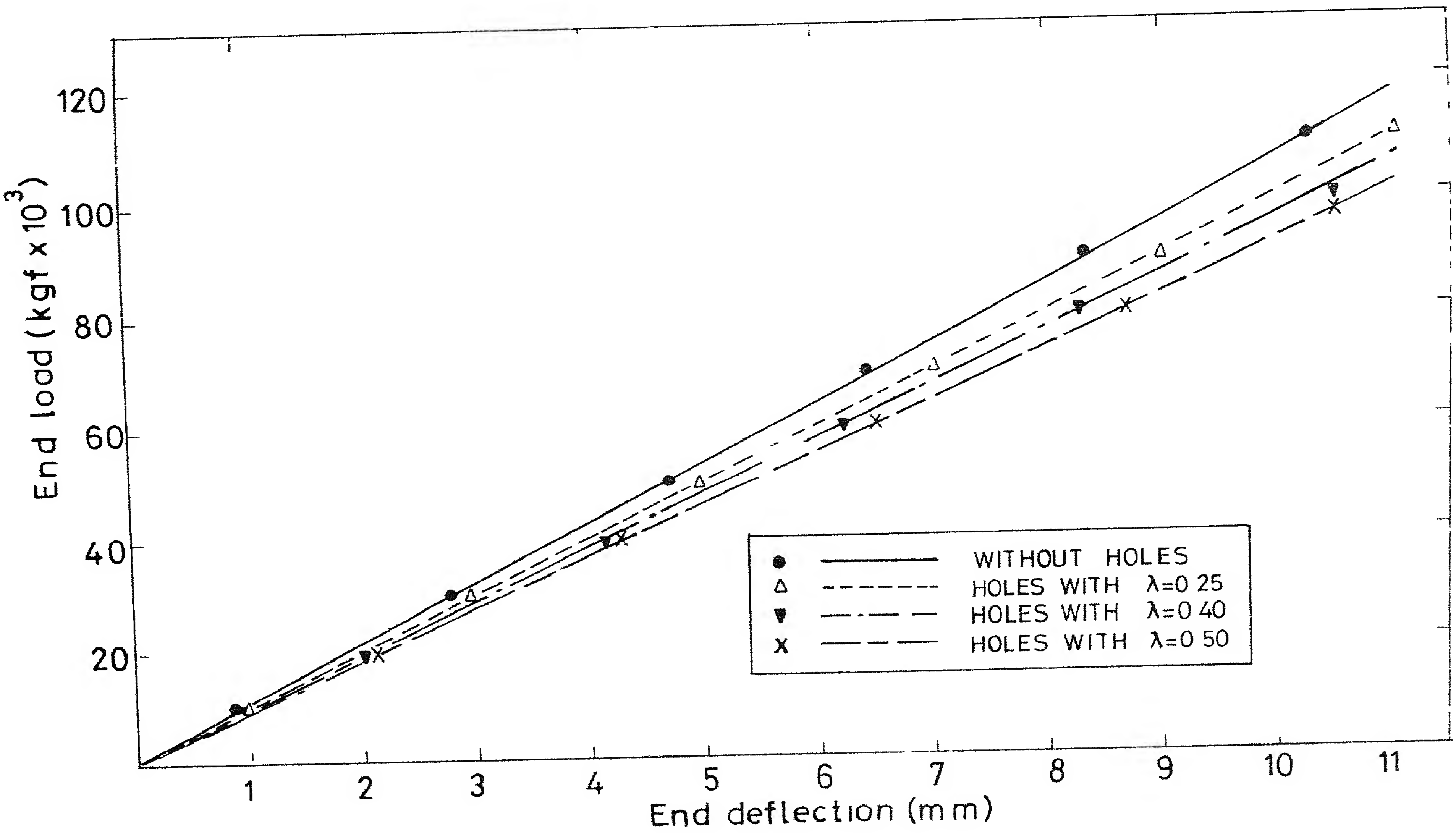
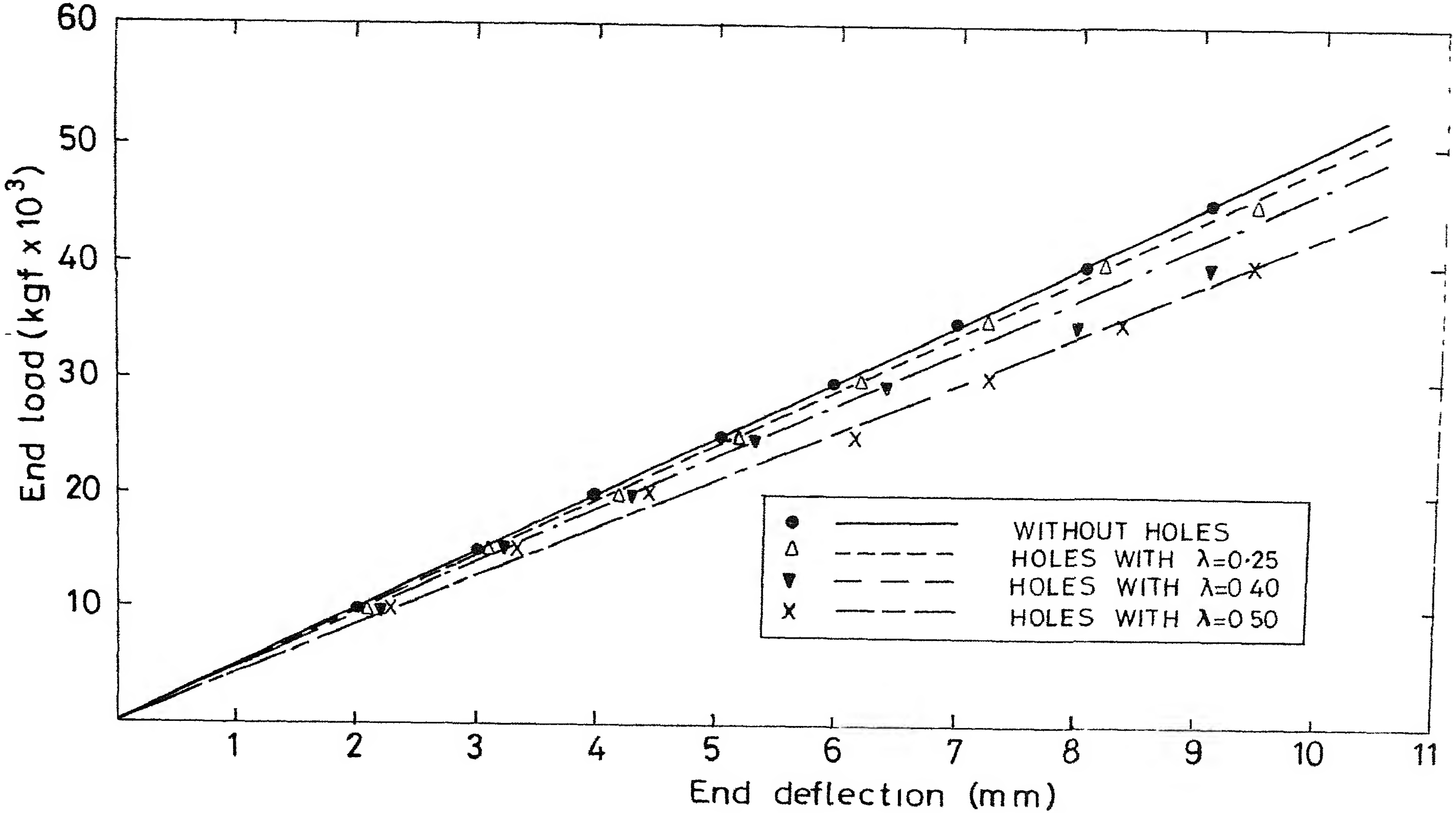


FIG 414 LOAD DEFLECTION CHARACTERISTICS OF CAST IRON SPECIMEN WITH DIFFERENT HOLE SIZES



FIG, 4·15 LOAD DEFLECTION CHARACTERISTICS OF BAKELITE SPECIMEN WITH DIFFERENT HOLE SIZES

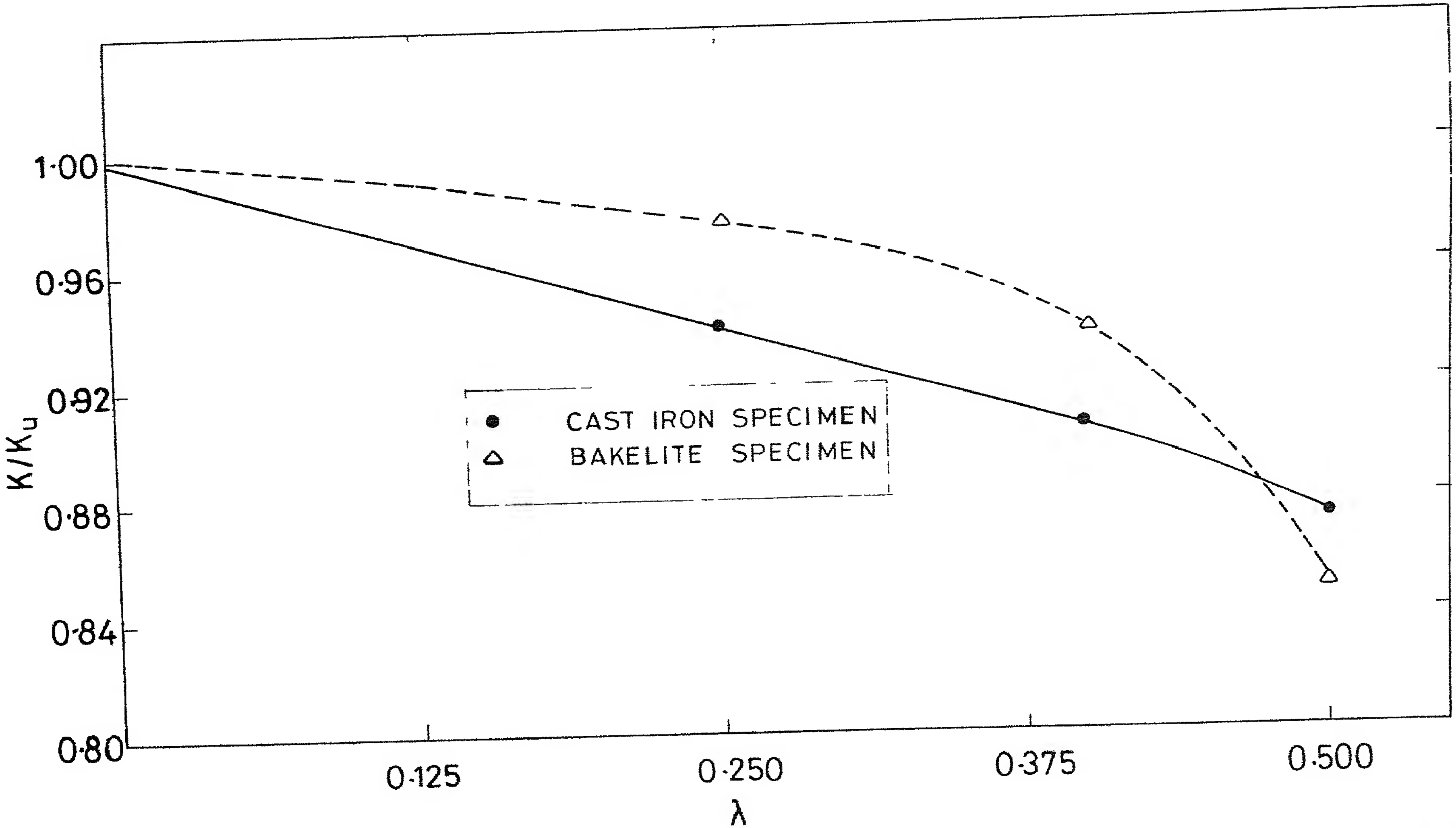


FIG 4 16 VARIATION IN STATIC RIGIDITY WITH DIFFERENT HOLE SIZES

4.4 Improvement of Damping Characteristics of Aluminium Specimen using High Damping Inserts

As discussed earlier, inserts of three different materials (Cast Iron, Bakelite and Perspex) were pressed into three Aluminium specimens. After recording the vibration decay of these specimens, the inserts were made annular as described in Chapter 3. Log-log plots of δ vs ε_m obtained from the records of these specimens are shown in Figs. 4.20 to 4.22. Improvement of damping capacity of the Aluminium specimen with inserts is shown in Figs. 4.17 to 4.19.

Before the detailed discussion of these results is presented, it should be noted that improvement in damping capacity with inserts is resulted due to one or more of the following reasons:

(a) contribution from the volume of the inherently high damping insert material,

(b) contribution from the stress concentration in the insert material which will be somewhat higher in case of annular inserts than that with solid inserts [17],

(c) contribution from the stress concentration in the parent material (Aluminium) itself, which will also be more with annular inserts than with solid inserts.

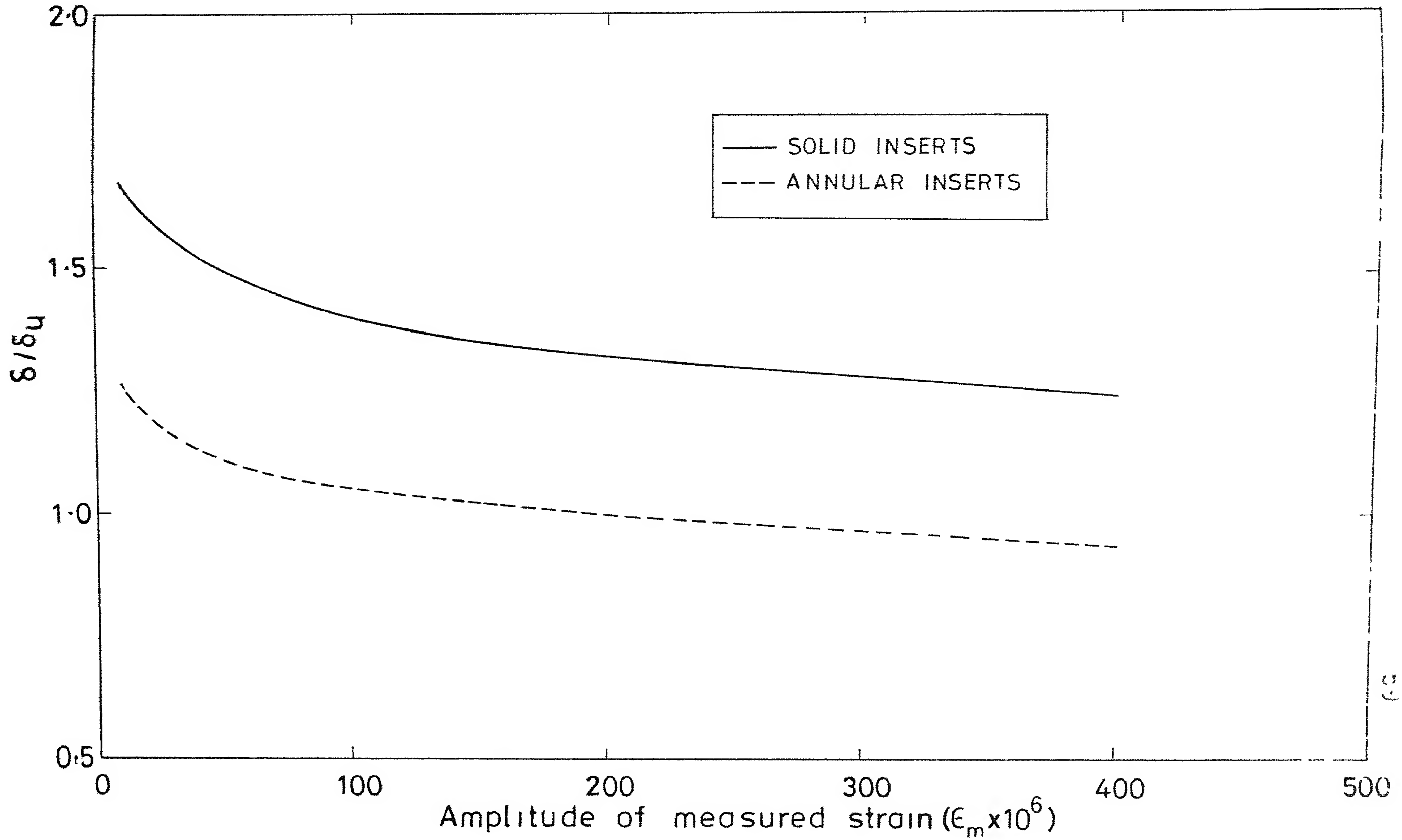


FIG. 4·18 IMPROVEMENT OF DAMPING CAPACITY OF ALUMINIUM SPECIMEN WITH BAKELITE INSERTS

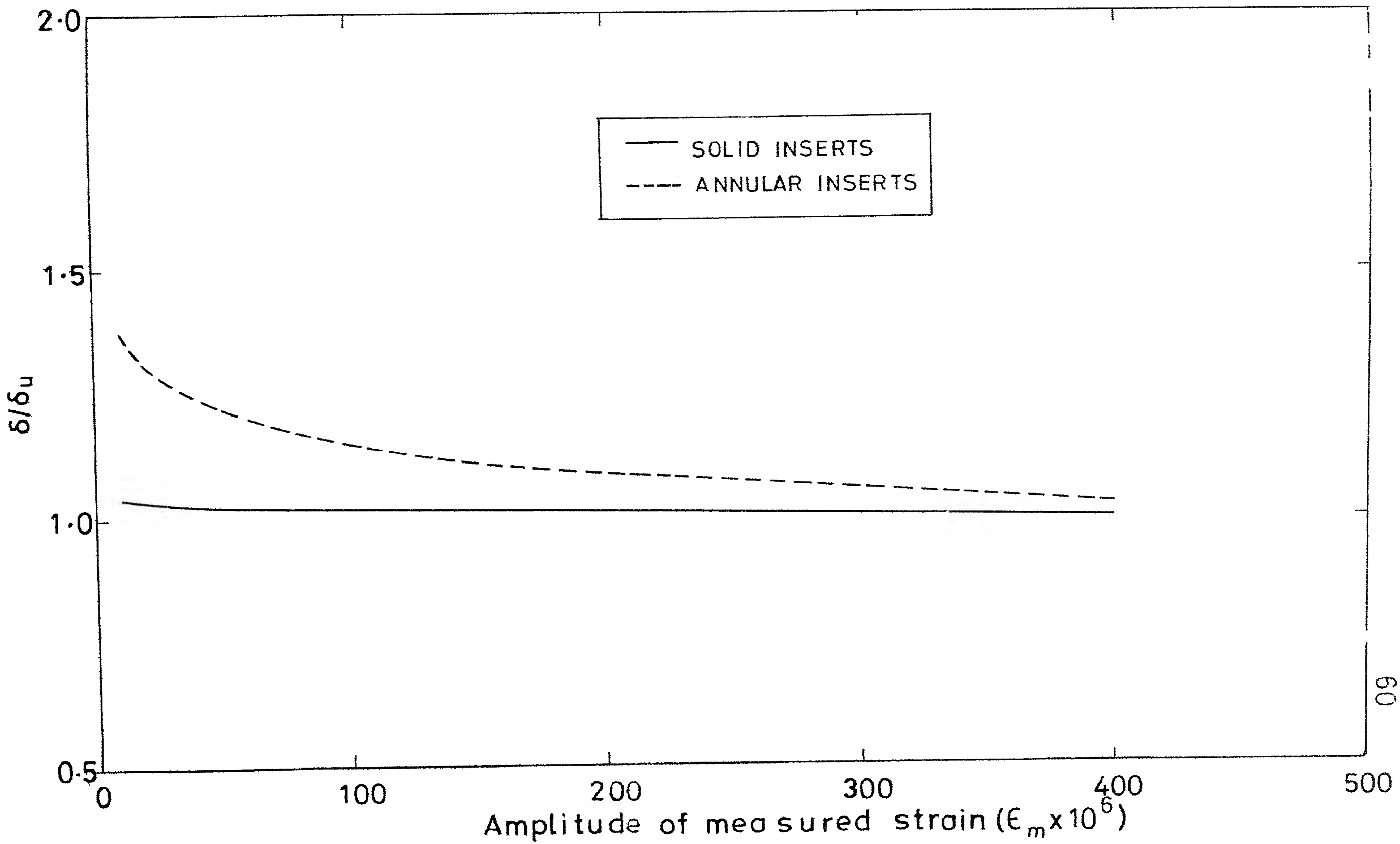


FIG. 4·19 IMPROVEMENT OF DAMPING CAPACITY OF ALUMINIUM SPECIMEN WITH PERSPEX INSERTS

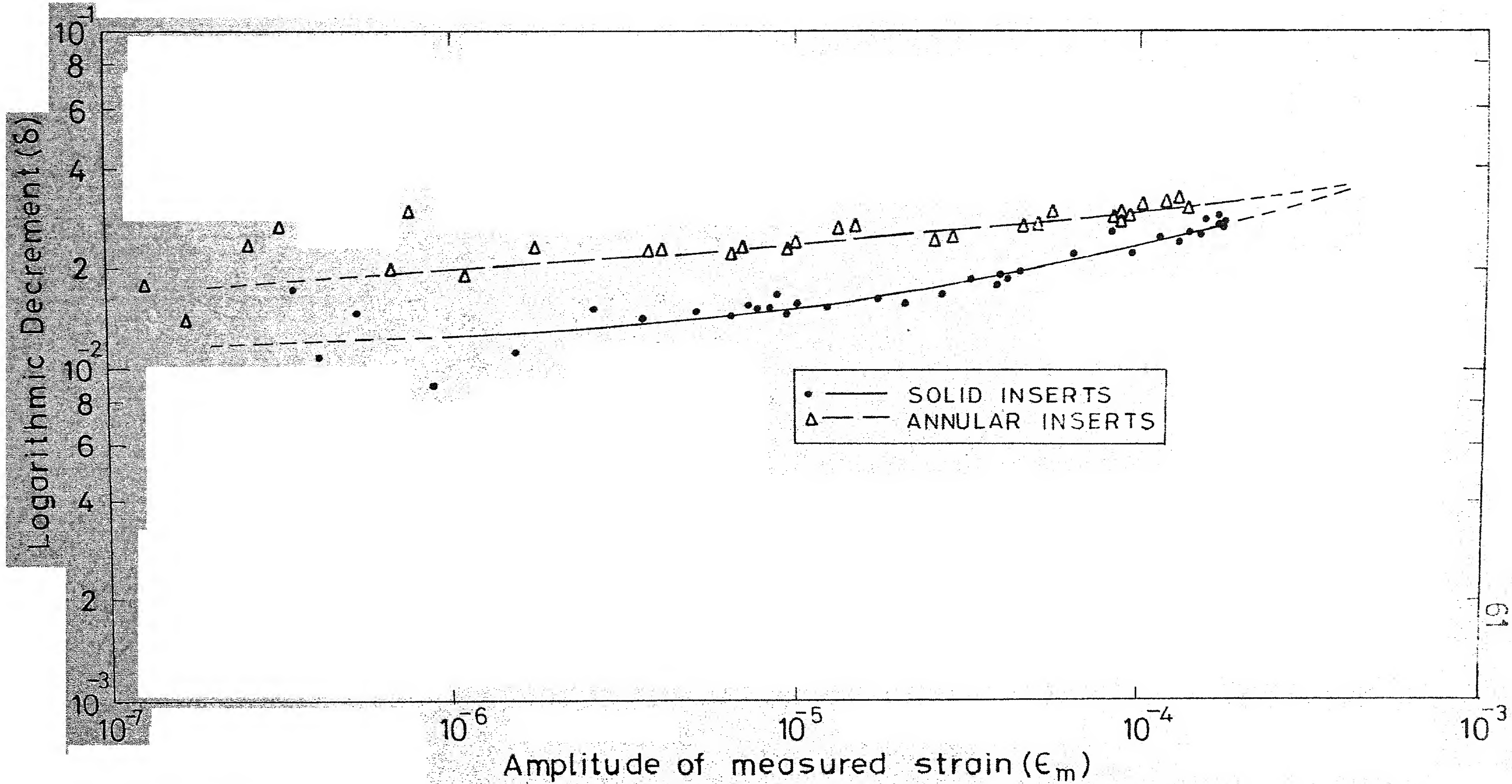


FIG. 4.20 DAMPING OF ALUMINIUM SPECIMEN WITH CAST IRON INSERTS

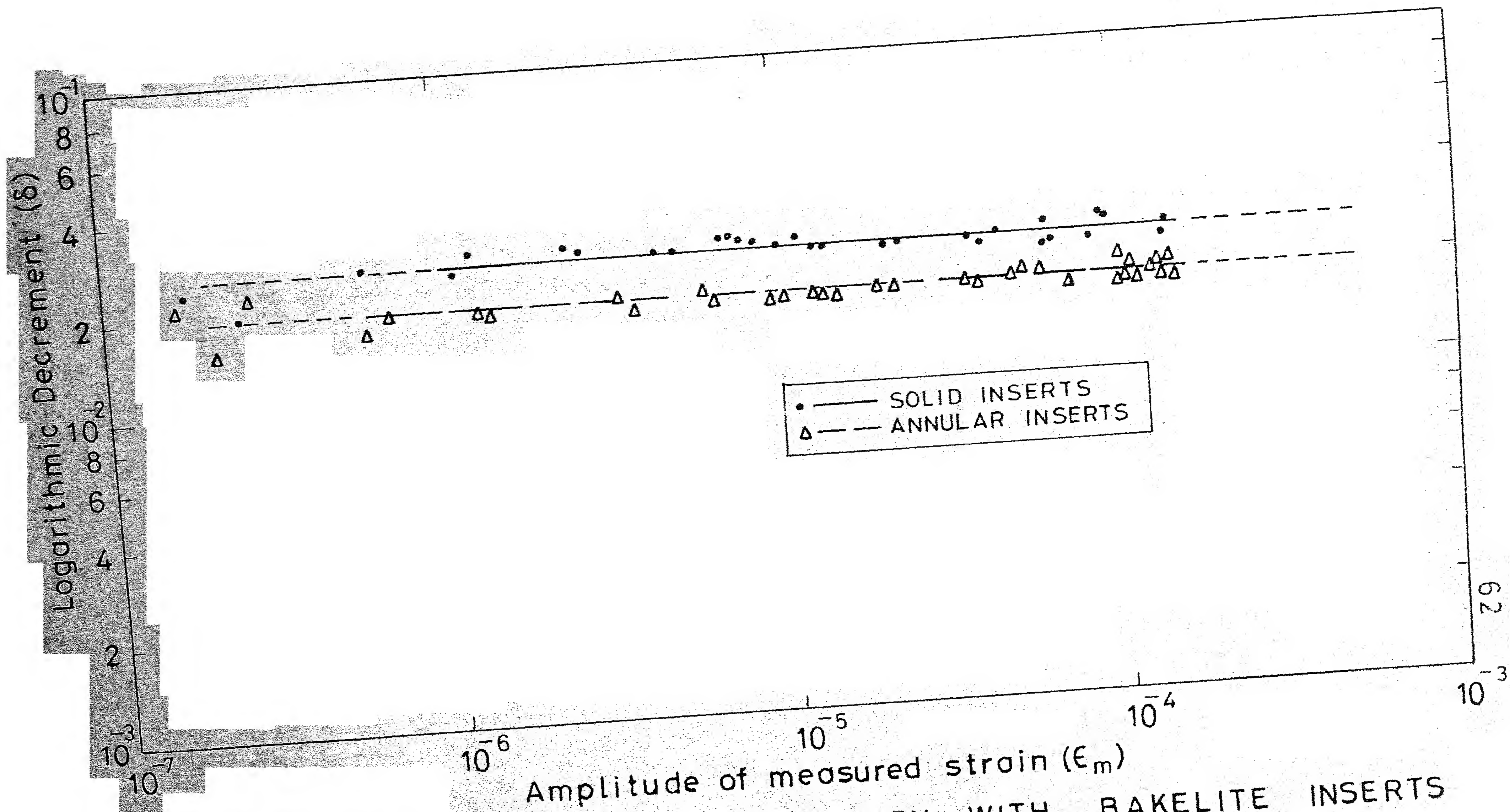


FIG. 4·21 DAMPING OF ALUMINIUM SPECIMEN WITH BAKELITE INSERTS

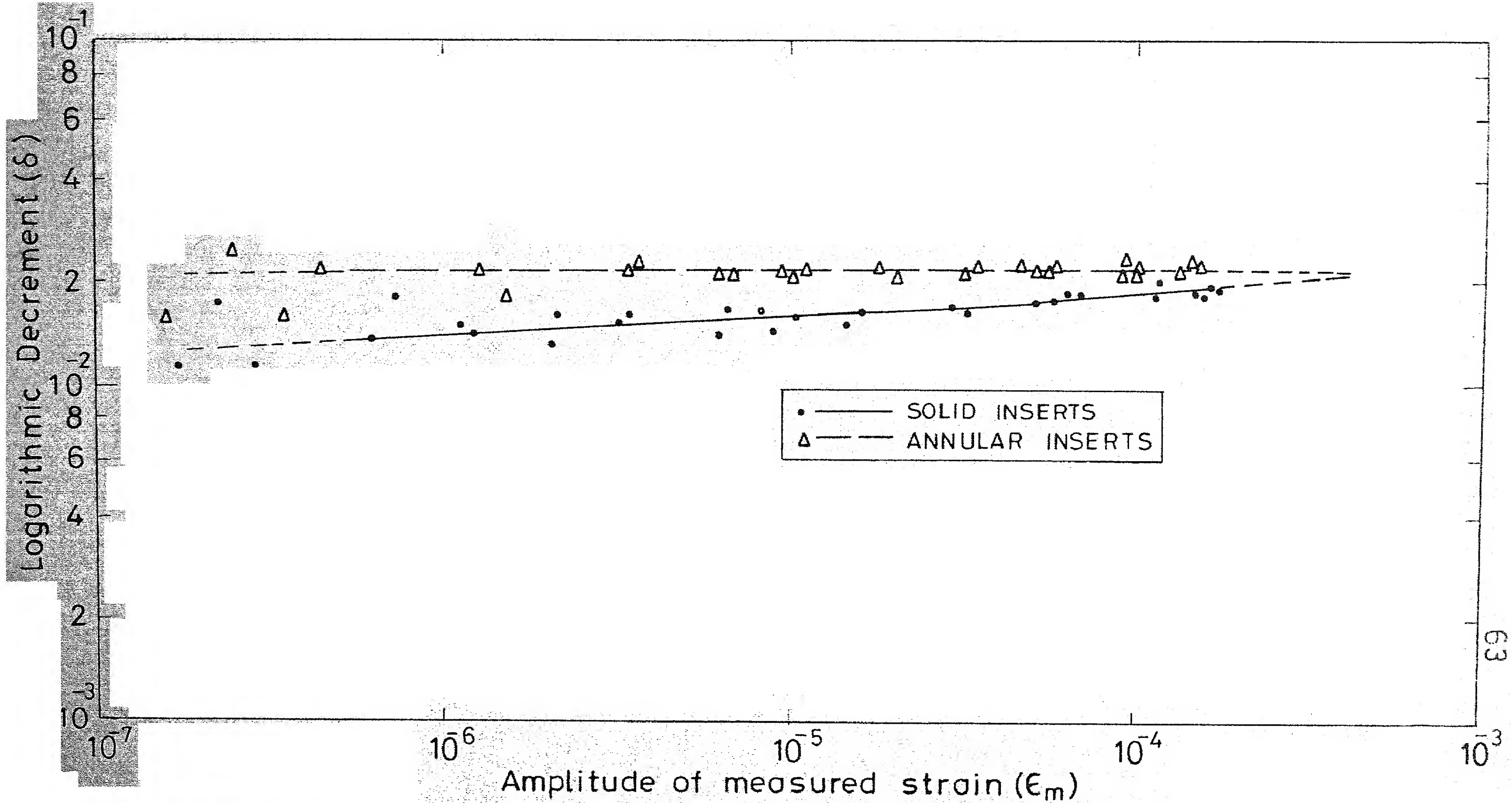


FIG.4.22 DAMPING OF ALUMINIUM SPECIMEN WITH PERSPEX INSERTS

Furthermore, the relative amount of stress concentration in the insert material and Aluminium depends on their relative moduli of elasticity. Moreover, it has already been seen in Sec. 4.3, that the effect of stress concentration towards improvement of damping is predominant at lower values of measured strain.

In reference [7] it has also been shown that the nature and the amount of improvement in the damping capacity depends on the type of fit. The following two extreme cases were considered

(a) welded inserts, and

(b) press-fit inserts with zero interference.

It was found that annular inserts of certain size produce maximum improvement in damping capacity in the case of welded inserts, whereas press-fit inserts yield maximum improvement when these are solid.

The actual fit for the inserts obtained during the experiments will be a case in between these two extremes. Since so many factors are simultaneously involved, no attempt has been made to pin-point the exact reasons for the improvement in the overall damping capacities of the specimens with various inserts.

(i) Cast Iron inserts: Table 4.1 shows that the values of J''s and n_2 of Cast Iron are much greater than those

of Aluminium. Hence it can be expected that both, volume of insert material and stress concentration in the insert (and the specimen), will contribute to damping improvement. From Fig. 4.17 it can be seen that at low values of the measured strain, annular inserts exhibit more damping improvement as compared to that by solid inserts. This may be attributed to the fact that stress concentration is more with annular inserts. However, at higher levels of the measured strain, the improvement is almost the same as that produced by solid inserts.

Average increment in the damping capacity of Aluminium specimen with Cast Iron inserts is seen to be of the order of 65% with annular inserts and 35% with solid inserts.

(ii) Bakelite inserts: Figure 4.18 shows that considerable improvement in the damping capacity of the Aluminium specimen is obtained with solid Bakelite inserts. Annular inserts give improvement only at low levels of the measured strain. This improvement is lesser than that obtained by solid inserts for the same level of measured strain. At higher levels of measured strain, annular inserts decrease the damping capacity of the specimen. The values of J''s of Bakelite are higher than those of Aluminium. There is not much difference in the values of n_2's of the two materials. Hence volume of the insert material rather

than stress concentration should contribute towards improvement in the damping capacity. Moreover, since the modulus of elasticity of Bakelite is lesser than that of Aluminium, stress values in the inserts are not high enough to contribute significantly towards damping improvement. Average improvement in the damping capacity with solid inserts is of the order of 45%.

(iii) Perspex inserts: The modulus of elasticity of Perspex is very small as compared to that of Aluminium. So any improvement in damping capacity due to stress concentration will be due to the stress concentration introduced in the Aluminium. The value of J_1' of Perspex (from Table 4.1) is seen to be less than twice that of Aluminium. This indicates that there will be marginal contribution towards improvement of damping due to the presence of the insert material. Figure 4.19 shows the improvement of the damping capacity of Aluminium specimen with Perspex inserts. It can be seen that there is little improvement in damping capacity with annular inserts; whereas there is almost no increase in the damping capacity with solid inserts. Average improvement in the damping capacity with annular inserts is about 20%.

The load-deflection characteristics of Aluminium specimens, with and without inserts, are shown in Fig. 4.23. It is seen that no significant change in the static rigidity takes place with either solid or annular inserts used in the present work.

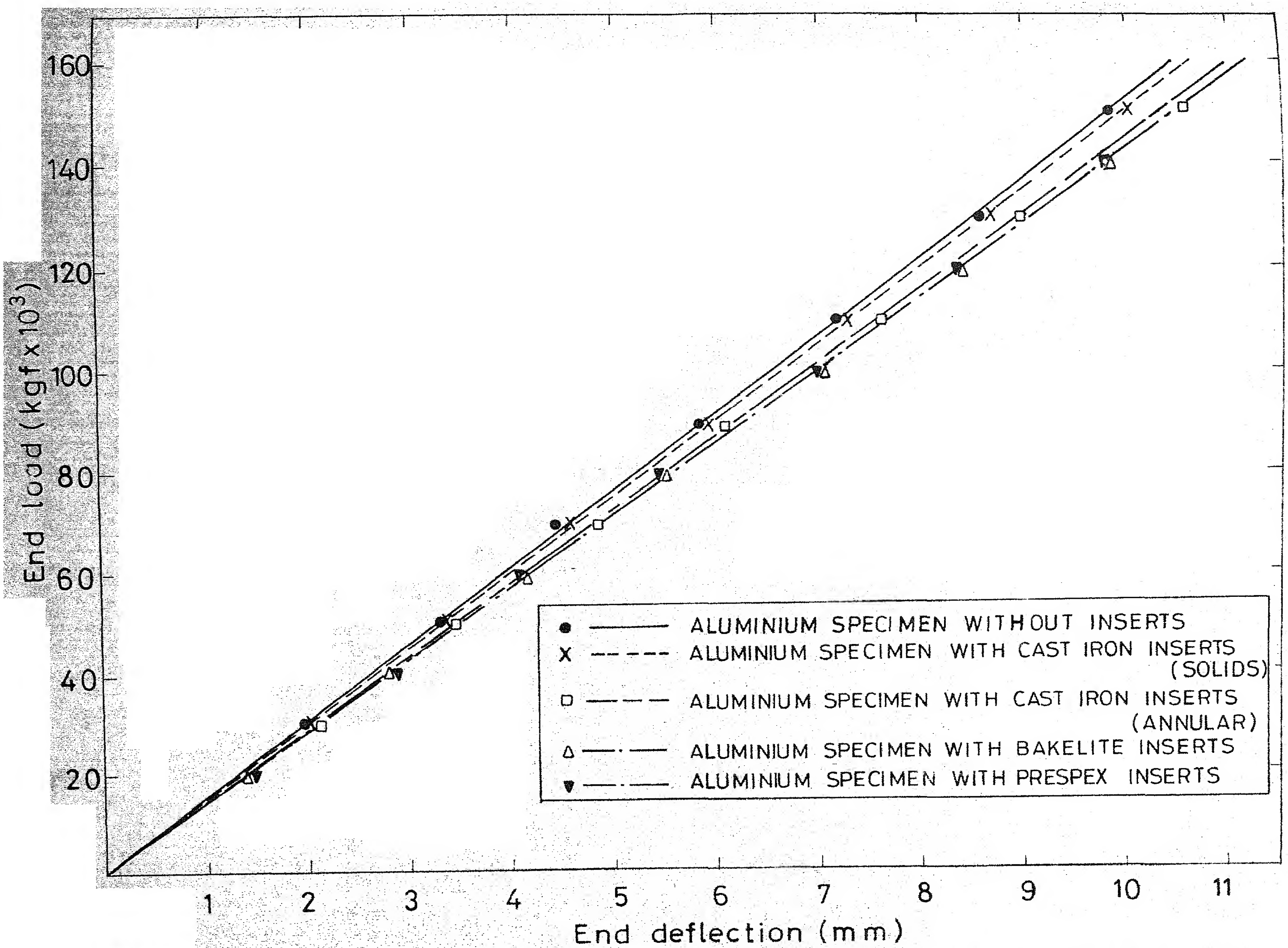


FIG. 4.23 LOAD DEFLECTION CHARACTERISTICS OF ALUMINIUM SPECIMEN WITH AND WITHOUT INSERTS

CHAPTER 5

CONCLUSIONS

The following conclusions can be drawn from the results of the present work.

(i) A test-rig, capable of measuring accurately damping capacities of materials has been developed.

(ii) Even at low frequencies of oscillation, (of the order of 10 Hz), air damping needs to be accounted for while measuring damping capacities of low damping materials like Aluminium.

(iii) Within the range of strain level covered in the present work, Perspex behaves linearly whereas Bakelite and Aluminium show non-linearity only near the higher end of this range. Cast Iron shows predominant non-linear behaviour even at low values of the strain amplitude.

(iv) Introduction of stress concentration increases the damping capacity of the Bakelite specimen by approximately 45% with an optimum hole size ($\lambda = 0.4$). The consequent fall in static rigidity is negligible.

(v) The effect of stress concentration in improving the damping capacity, is found to be more predominant at lower values of the amplitude of oscillation.

(vi) In the case of the Cast Iron specimen, there is no significant improvement in the damping capacity with introduced stress concentration.

(vii) Cast Iron and Bakelite inserts increase the damping capacity of Aluminium specimen considerably with almost no change in the static rigidity. No significant improvement is observed with Perspex inserts.

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